Math 141- Spring 2004 _____ Name:_____ANSWERS_____

Test I (chapters 1 and 2) 100

Show all your work!! Partial credit is based on work shown!! Seat:_____

6pts

1. Fill in the next two numbers for each sequence:

a. 6, 18, 54, 162, 486, ____1458_____, ___4374_____

b. 4, 10, 16, 22, 28, 34, ____40_____, ____46_____

c. 4, 16, 64, 256, 1024, _____4096____, ____16384_____

8pts

2. For this sequence from problem 1b 4, 10, 16, 22, 28, 34, **. . .**

a. Describe the sequence **with words:**

Beginning with 4, add 6 to get the next number in the sequence.
Continue this process, adding 6 to any number to get the next number in the sequence.

b. Describe this sequence with a **formula** using n as the variable; that is, what is the formula that would generate the sequence if n = 1, then n = 2, etc.?

Repeated adding of 6 makes each number in the sequence = [the original #] plus a multiple of 6.
That is, [4 + (a multiple of) 6]

[4 + (n - 1) 6] or this also could be written as [6n – 2]

c. What is the 201st term of this sequence?

4 + (201 – 1) 6 = 4 + (200) 6 = 1204

10pts

3. a. Pick any number and try the following "number magic".

		1st try	2nd try	algebraic proof
Pick a number	Examples:	5	8	n
Subtract 2		3	6	n - 2
Multiply by 4		12	24	4n - 8
Add 14		26	38	4n + 6
Subtract the original number		21	30	3n + 6
Divide by 3		7	10	n + 2
(Show your result)				

b. What generalization can you make about <u>how the result is related to each number picked</u>?

The result is two more than the original number picked.

c. Use algebra (in space **above)** to prove that your generalization is correct. _____

Notice the result is (n + 2) if n is the original number picked.

6pts

4. In chapter 1, we studied strategies for planning how to solve problems. List six of these strategies.

See page 4 in textbook!

Guess and test	Look for a pattern	Inductive Reasoning
Draw a picture	Use a variable	Deductive Reasoning
Make a list	Solve a simpler problem	Use properties of numbers (odd, even, etc)

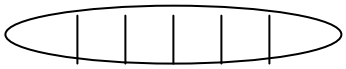
_____

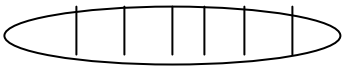
30pts

14pts

5. **Solve** each of the following problems, **showing your reasoning and calculations**, then **list the problem solving strategies** (see problem #4 above) that you used.

a. How many cuts does it take to divide a log into:
six equal cross-sectional pieces?__5____ seven equal cross-sectional pieces?___6___





How many cuts does it take to divide a log into "n" equal cross-sectional pieces?___(n – 1) ___

List the strategies you used: Draw a picture, look for a pattern, inductive & deductive reasoning.

b. There is an old riddle about a worm at the bottom of a 28-foot well. If the worm climbs up 4 feet each day and slips back 1 foot each night, how many days will it take him to climb out of the well?

The worm climbs up 4 feet and falls back 1 foot each day and thus gains 3 feet a day.
But on the last day, it climbs up 4 feet and gets out and does not fall back.
Thus it takes 9 days for the worm to get out.

3 feet for each of 8 days = 24 feet, then on the 9th day it climbs 4 feet and gets out.

List the strategies you used: Look for a pattern, deductive reasoning and could draw a diagram.

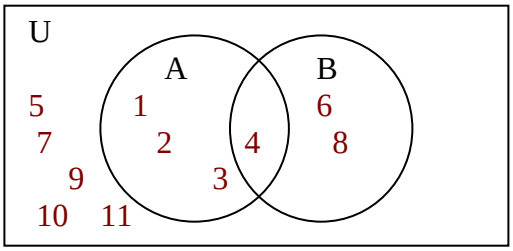
14pts

6. If A = {1, 2, 3, 4} and B = {4, 6, 8} and the universal set, U = {x | x ϵ whole numbers, 0 < x < 12} then:

a. A ∪ B = __ {1, 2, 3, 4, 6, 8 }____

b. A – B = ____{1, 2, 3 }_____

c. List the elements in the appropriate parts of this Venn Diagram for sets A, B and U:



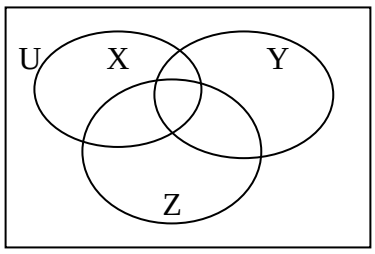
d. The Cartesian Product (A X B)
has **how many** elements? __12____
n(A) = 4 , n(B)= 3 so n(A X B) = 4 x 3 = 12

e. List **all** the subsets of set B:
{4} {4, 6} { }
{6} {4, 8} {4, 6, 8}
{8} {6, 8}

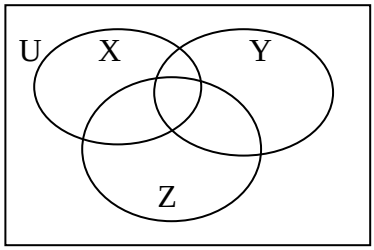
4pts

7. Shade this Venn diagram to represent the sets: I can't shade these so I have described the answers.

a. $\overline{(X \cap Y)}$



b. $[\, Z \cap \overline{(X \cap Y)}\,]$



_____
32pts

a. Everything should be shaded except for the intersection of set X with Y.
b. Only shade the part of set Z that intersects with the answer in part a.

8pts

8. **True or False:** (If false, tell why it is false or correct the statement.)

a. ___True_____ {1, 2, 3 } ~ {x, y, z} Sets are equivalent if they have the same number of elements.

b. ___False_____ If X ⊂ Y, then X ∩ Y = Y. Should be: If X ⊂ Y, then X ∩ Y = X.

c. ___False_____ If 8 ∈ {C ∩ D}, then 8 is in either set C or set D. Should be: set C AND set D.

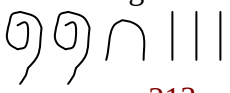
d. ___False_____ A ∪ { } = { } Should be: A ∪ { } = A or could be: A ∩ { } = { }

6pts

9. Write the usual Hindu-Arabic numeral for each of the following numerals:

a. MCDLXXVIII

1478

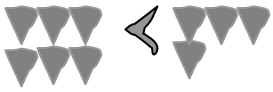
b. 

213

6pts

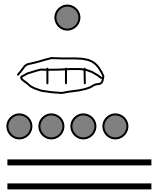
10. Write 374 in each of the following number systems.

a. Babylonian



6 (60) + 14 (1)

b. Mayan



1 (18 x 20) + 0 (20) + 14 (1)

6pts

11. a. 374_{ten} = ____1422____$_{six}$

Divide by powers of 6 to regroup numbers.

$1(6^3) + 4(6^2) + 2(6^1) + 2(6^0)$
$1(216) + 4(36) + 2(6) + 2(1)$
1422

b. 312_{four} = ____54____$_{ten}$

Rewrite in expanded notation for base four.
Then multiply and add to convert to base ten.

$3(4^2) + 1(4^1) + 2(4^0)$
$3(16) + 1(4) + 2(1)$
48 + 4 + 2
54

4pts

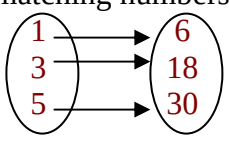
12. If you are counting in base three, fill in the blanks to show what numbers would follow these:

1, 2, 10, 11, 12, 20, _21_, _22_, __100__, _101_, _102_, _110__, _111_, _112___

8pts

13. A particular function is the matching of a whole number with its multiple of 6. This could be expressed with the formula y = 6n. If the domain of the function is n ={1, 3, 5}, what is the range of the function? ___{ 6, 18, 30 }___ Express this function in each of the following ways:

a. As an arrow diagram. (matching numbers in two sets)



b. As a set of ordered pairs (n , y)

{ (1, 6), (3, 18), (5, 30) }

c. As a function machine

see pictures in textbook

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38pts