

Answers to Sample Test

Math 141 - Summer 2004
 Test I (chapters 1 and 2) 100
Show all your work!!

Name: Key, July 7, 2004
 Seat: _____

Partial credit is based on work shown!!

6pts (1pt each blank)

1. Fill in the next two numbers for each sequence:

a. $4^1, 4^2, 4^3, 4^4, 4^5$
 4, 16, 64, 256, 1024, 4096, 16384

b. $6^3, 18^3, 54^3, 162^3, 486^3$
 6, 18, 54, 162, 486, 1758, 4374

c. $4, 11, 18, 25, 32$
 4, 11, 18, 25, 32, 39, 46

8pts

2. For this sequence from problem 1c 4, 11, 18, 25, 32, ...

a. Describe the sequence **with words**.

Begin with 4 and add 7 to get the next #. Continue this process adding 7 to any # to get the next #. 2pts

b. Describe this sequence with a **formula** using n as the variable; that is, what is the formula that would generate the sequence if $n = 1$, then $n = 2$, etc.?

3pts

$4 + (n-1)7$ or $7n - 3$

c. What is the 301st term of this sequence?

2pts

$4 + (300)7 = 2104$ or $7(301) - 3 = 2107 - 3 = 2104$

10pts

3. a. Pick any number and try the following "number magic".

	1st try	2nd try	algebraic proof	
Pick a number	2	7	x	\square
Add 5	7	12	$x+5$	$\square+5$
Multiply by 4	28	48	$4x+20$	$\square\square\square\square+20$
Subtract the original number	26	41	$3x+20$	$\square\square\square+20$
Subtract 2	24	39	$3x+18$	$\square\square\square+18$
Divide by 3	8	13	$x+6$	$\square+6$

b. What generalization can you make about how the result is related to each number picked?

The result is 6 more than the original # picked. 2pts

c. Use algebra (in space above) to prove that your generalization is correct.

6pts

4. In chapter 1, we studied strategies for planning how to solve problems. List six of these strategies.

see p. 4

- Guess + test
- Use a variable
- Draw a picture
- make a list (or table)
- Look for a pattern
- Solve a simpler problem (to find pattern)

- Inductive Reasoning.
- Deductive reasoning or $\left\{ \begin{array}{l} \text{indirect reasoning} \\ \text{direct reasoning} \end{array} \right.$
- work back wards
- Role play - act out the problem
- Number properties

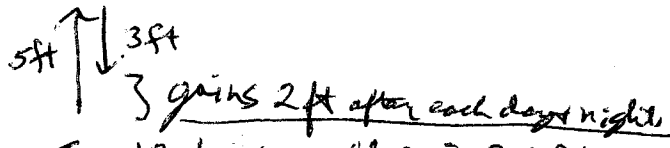
7pts for each word problem } 3pts work shown
 2pts answer
 2pts strategies listed

14pts

5. Solve each of the following problems, showing your reasoning and calculations, then list the problem solving strategy (see problem #4 above) that you used.

a. A well is 30 feet deep. A worm climbs up 5 feet each day and then slides back 3 feet each night. How many days will it take the worm to crawl out of the well?

14 days.
 -1pt if 15 days



In 13 days would gain 26 ft,
 So on 14th day (26+5=31) would crawl out.

List the strategies you used:
 Draw a picture, see a pattern

b. Together a baseball and a football weigh 1.25 pounds, the baseball and a soccer ball weigh 1.35 pounds, and the football and the soccer ball weigh 1.9 pounds. How much does each of the balls weigh?

$$\begin{aligned} b + f &= 1.25 \\ b + s &= 1.35 \\ f + s &= 1.9 \end{aligned}$$

subtract $s - f = 0.10$
 add $s + f = 1.9$

 $2s = 2.0$

$s = 1 \text{ lb.}$
 Thus $b = 0.35 \text{ lb.}$
 and $f = 0.9 \text{ lb.}$

f	s	b
1	1.1	.25
:	:	:
.9	1	.35

List the strategies you used:

14pts

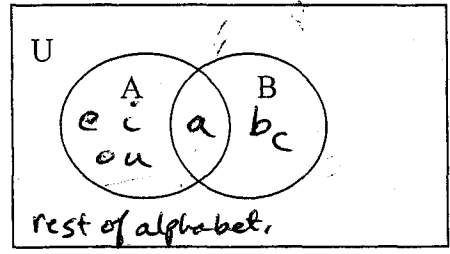
Used a variable, Use equations, Make a list, guess + test.

6. If $A = \{a, e, i, o, u\}$ and $B = \{a, b, c\}$ and the universal set, $U = \{x \mid x \in \text{the English alphabet}\}$ then:

a. $A \cup B = \{a, b, c, e, i, o, u\}$ 2pts

b. $B - A = \{b, c\}$ 2pts

c. Draw a Venn diagram, listing the elements in each part of each set (for sets A, B & U.) 4pts

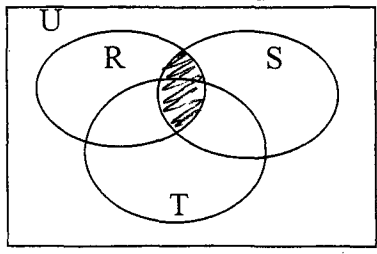


d. The Cartesian Product $A \times B$ has how many elements? 15 2pts

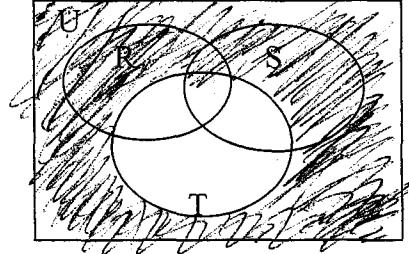
e. List all the subsets of set B: $\{a, b, c\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{a\}$, $\{b\}$, $\{c\}$, $\{\}$ 4pts

7. Shade each Venn diagram to represent the set.

a. $(R \cap S)$ 2pts



b. \bar{T} 2pts



-1pt if small error

8pts

8. True or False: If false, tell why it is false or correct the statement.

a. true $(A \cap B) \subseteq (A \cup B)$

b. true If $X \subset Y$, then $X \cap Y = X$.

$(X \cap Y)$

1* for each correction.

c. false If $5 \in \{C \cap D\}$, then 5 is in set C and in set D.

d. false $A \cup \{ \} = \{ \} \cup A$

6pts

9. Write the usual Hindu-Arabic numeral for each of the following numerals:

a. MCDLXXXIV

1484

b.

$\text{999} \text{nn} \text{llll} = \text{324}$

6pts

10. Write 232 in each of the following number systems.

a. Babylonian

$\text{VVV} \text{ <<< } \text{VV}$

b. Mayan

 ..

$$\begin{array}{r} 3 \\ 60 \overline{) 232} \\ \underline{180} \\ 52 \end{array}$$

$3(60) + 5(10) + 2(1)$

$$\begin{array}{r} 12 \\ 20 \overline{) 232} \\ \underline{200} \\ 32 \\ \underline{20} \\ 12 \end{array}$$

$11(20) + 12$

4pts

11. If you are counting in base three, fill in the blanks to show what numbers would follow the given ones. -1* for each smaller error.

1, 2, 10, 11, 12, 20, 21, 22, 100, 101, 102, 110, 111, 112, 120

6pts

12. a. $212_{\text{four}} = \underline{38}_{\text{ten}}$

b. $187_{\text{ten}} = \underline{511}_{\text{six}}$

$$\begin{aligned} & 2(4^2) + 1(4) + 2(1) \\ \text{or} & \underline{2(16) + 1(4) + 2(1)} \\ & 32 + 4 + 2 = \end{aligned}$$

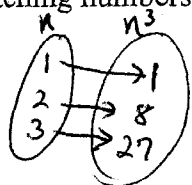
$$\begin{array}{r} 5 \\ 36 \overline{) 187} \\ \underline{180} \\ 7 \end{array} \quad \text{-1* } 4507_{\text{six}}$$

 $5(6^2) + 1(6) + 1(1)$

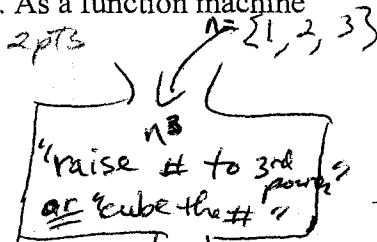
8pts

13. A particular function is the matching of a whole number with the cube of the number. This could be expressed with the formula $y = n^3$. If the domain of the function is $n = \{1, 2, 3\}$, what is the range of the function? $\{1, 8, 27\}$ Express this function in each of the following ways:

a. As an arrow diagram. 2pts b. As a set of ordered pairs 2pts c. As a function machine 2pts



$\{(1, 1), (2, 8), (3, 27)\}$



OK if just an example of one # going to one number
 $\frac{27}{3} = 9$
 $\frac{38}{3}$

$\rightarrow 1, 8, 27$