

Math 141 Problem Solving

Sherlock Holmes and Dr Watson went on a camping trip. After a good meal and a bottle of wine they lay down in their tent for the night, and went to sleep.

Some hours later, Holmes awoke and nudged his faithful friend awake.

"Watson, look up at the sky and tell me what you see." Watson replied, "I see millions and millions of stars."

"What does that tell you?" Holmes questioned. Watson pondered for a minute.

"Astronomically, it tells me that there are millions of galaxies and potentially billions of planets. Astrologically, I observe that Saturn is in Leo. Chronologically, I deduce that the time is approximately a quarter past three. Theologically, I can see that God is all powerful and that we are small and insignificant. Meteorologically, I suspect that we will have a beautiful day tomorrow. What does it tell you?"

Holmes was silent for a minute, then spoke.

"Watson, you idiot. Someone has stolen our tent."

When you are trying to solve a problem, don't think the answer must be something really complex, look for obvious solutions as well.

Solve the following problems:

1. A textbook is opened and the product of the page numbers of the two facing pages is 650. What are the numbers of the pages? (Repeat this problem where the product of the two facing pages is 6162.)
2. If possible, find an odd number that can be expressed as the sum of four consecutive counting numbers. If impossible, explain why.
3. Place 10 stools along four walls of a room so that each of the four walls has the same number of stools.
4. If the diagonals of a square are drawn in, how many triangles of all sizes are formed?
5. The distance around a standard tennis court is 228 feet. If the length of the court is 6 more than twice the width, find the dimensions of the tennis court.

Number Puzzles

Many magic tricks and “mind-reading” activities can be explained through the use of simple algebraic techniques. Consider as an example the “think of a number” type of mathematical activity.

- Think of a number.
- Add 3 to this number.
- Multiply your answer by 2.
- Subtract 4 from your answer.
- Divide by 2.
- Subtract the number with which you started.

If you follow these instructions carefully, your answer will always be 1, regardless of the number with which you start. Using algebraic symbols or drawing pictures, as shown below, can explain why this happens.

Think of a number:	n	<input style="width: 30px; height: 25px;" type="text"/>	
			(The empty box represents a variable since any number could be put inside the box.)
Add 3:	$n + 3$	<input style="width: 30px; height: 25px;" type="text"/> ○ ○ ○	
Multiply by 2	$2n + 6$	<input style="width: 30px; height: 25px;" type="text"/> ○ ○ ○ <input style="width: 30px; height: 25px;" type="text"/> ○ ○ ○	
Subtract 4	$2n + 2$	<input style="width: 30px; height: 25px;" type="text"/> ○ <input style="width: 30px; height: 25px;" type="text"/> ○	
Divide by 2	$n + 1$	<input style="width: 30px; height: 25px;" type="text"/> ○	
Subtract the original number, n :	1	○	

Trying this with a variety of different numbers, noticing that the answer is 1 each time, and concluding that the answer would always be 1, is an example of inductive reasoning.

Definition of Inductive Reasoning -- Look at a number of examples to find a pattern and make a generalization (hypothesis) based on this pattern. This is the "scientific method" you learned in general science classes. This does not prove that your hypothesis is true, it just makes it likely to be true based on the examples you observed.

Using algebra to prove the answer will always be 1 is an example of deductive reasoning.

Definition of Deductive Reasoning -- Uses algebra or steps in logical reasoning based on facts, which allows you to prove a hypothesis is true.

(In the list of strategies in your textbook these are named indirect reasoning and direct reasoning.)