Outline of Math 141 Test I (Expanded version)

(Created by Robert R. Ferris, summer 2005, revised and updated by Karen E. Spike)
Chapters 1 and 2

1. Four steps in problem solving & nine strategies for making a plan in problem solving. See page four.

The four steps and nine of the strategies for making a plan in problem solving are:

Step1: Understand the problem

- *Do you understand all the words?*
- *Can you restate the problem in your own words?*
- *Do you know what is given?*
- *Do you know what the goal is?*
- *Is there enough information?*
- *Is this problem similar to another problem you have solved?*

Step 2: Devise a Plan

1. *Guess and Test*
2. *Use a variable*
3. *Draw a picture*
4. *Look for a pattern*
5. *Make a list*
6. *Solve a simpler problem*
7. *Draw a diagram*
8. *Use direct reasoning*
9. *Use indirect reasoning*
10. *Use properties of numbers*

Step 3: Carry Out the Plan

- *Implement the strategy or strategies until the problem is solved or until a new course of action is suggested.*

- *Allow a reasonable time to solve the problem. If not successful look for hints from others or set the problem aside for a while.*

- *Don't fear starting over. Often a fresh start and a new strategy is the winning combination*

Step 4: Look Back

- Is your solution correct? Does your answer satisfy the statement of the problem?
- Can you see an easier solution?
- Can you see how you can extend you solution to a more general case?

2. Solve 2 word problems from chapter 1. Show all work, clearly state answer, and list the strategies used.

3. "Number magic" -- See class notes, Activity 1 from module 1, with sample problem and solution done with variable and an "open box" for a variable.

Pick any number	***n***	or	□
Add three	***n +3***	or	□ ○ ○ ○
Multiply by two	***2n + 6***	or	□ □ ○○ ○○ ○○
Subtract four	**2n + 2**	or	□ ○ ○
Divide by two	**n + 1**	or	□ ○
Subtract the original number			
The result is	**1**	or	○

<u>Trying this with a variety of different numbers</u>, noticing that the answer is 1 each time, and <u>concluding that the answer would always be 1</u>, is an example of <u>inductive reasoning</u>.
Definition of Inductive Reasoning -- Look at a number of examples to find a pattern and make a generalization (hypothesis), based on this pattern. This is the "scientific method" taught in general science classes. This does not prove that your hypothesis is true, it just makes it likely to be true based on the examples you observed.

<u>Using algebra to prove the answer will always be 1 is an example of deductive reasoning</u>.
Definition of Deductive Reasoning -- Uses algebra or steps in logical reasoning based on facts, which allows you to prove a hypothesis is true.
In the list of problem solving strategies in the text, these are listed as indirect and direct reasoning.

4. Sequences. Be able to extend a sequence, verbally describe the sequence, write a formula for the sequence, and calculate a particular term of the sequence. Section 1.2 and also section 2.4. (In the 9th edition text this is section 9.3.)

1. See a pattern and be able to extend the sequence, i.e.,
   5, 8, 11, 14, ____, ____, ____

2. Describe a pattern in words:
   The last number in the sequence is obtained by subtracting the first number from the second number to obtain the common difference. Then add to the first term, the common difference multiplied by one less than the term being obtained.

$$n^{th}\ \# = 1^{st}\ \# + (n - 1)(\text{common difference})$$

or $$n^{th}\ \# = 1^{st}\ \# + (n - 1)(\text{number added})$$

3. Describe using a formula

n^{th} Term = $a_n = a_1 + (n - 1)\,d$

Find 201^{st} term $5 + (201-1)\,3$

$5 + (200)3 = 5 + 600 = 605$

Therefore, the 201^{st} term in the sequence is 605.

4. Be able to find a specific term in the sequence without writing the entire sequence,

	Evens			Odds	
n	***f(n)= 2n or f(n)=2+(n-1)2***			***g(n) = 2n-1 or g(n) = 1+(n-1)2***	
1	**2**	2		**1**	1
2	**4**	2 + 1(2)		**3**	1 + 1(2)
3	**6**	2 + 2(2)		**5**	1 + 2(2)
4	**8**	2 + 3(2)		**7**	1 + 3(2)
5	**10**	2 + 4(2)		**9**	1 + 4(2)
6	**12**	2 + 5(2)		**11**	1 + 5(2)
7	**14**	2 + 6(2)		**13**	1 + 6(2)
20	*40*	***2 + 19(2)***		***21***	***1 + 10(2)***

n^{th} Term = $5 + (n - 1)\,3$

$a_n = a_1 + (n - 1)\,d$

n	***f(x) = 2n***	
1	**5**	5
2	**8**	5 + 1(3)
3	**11**	5 + 2(3)
4	**14**	5 + 3(3)
5	**17**	5 + 4(3)
6	**20**	5 + 5(3)
7	**23**	5 + 6(3)
101	***305***	***5 + 100(3)***

=305

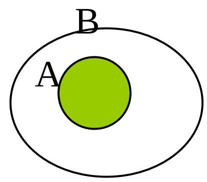
Find 101^{st} term $5 + (101-1)\,3$

$5 + (100)3 = 5 + 300 = 305$

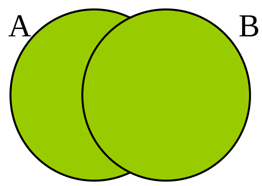
Therefore, the 101^{st} term in the sequence is 305.

5. Sets: Section 2.1. Understanding of all basic concepts: subset, universal set, intersection, union, complement, set difference, Cartesian product, and use of Venn diagrams.

Subset $A \subseteq B$
"Set A is contained in set B"

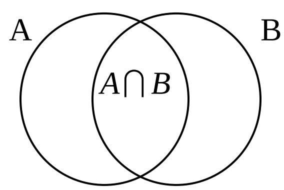


Union of sets, A U B
"combination of A and B"



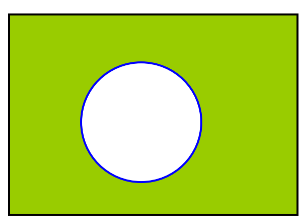
$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

Intersection of sets, $A \cap B$
"overlap of A and B"



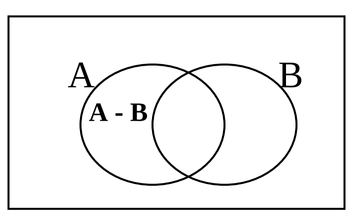
$A \cup B = \{x \mid x \in A \text{ and } x \in B\}$

Complement of a set, $\bar{\mathbf{A}}$
"elements in U but not in set A"



$\bar{A} = \{x \mid x \in U \text{ and } x \notin A\}$

Set Difference, A- B
"elements in A but not in B"



$A - B = \{x \mid x \in A \text{ and } x \notin B\}$

Cartesian Product or Cross Product, A x B
"ordered pairs of elements from sets A and B"

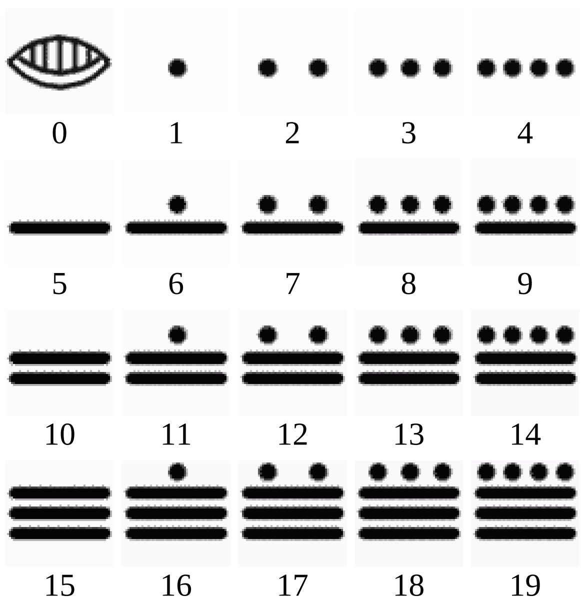
$A \ X \ B = \{(a,b) \mid a \in A \text{ and } b \in B\}$

If A={r, w, b} and B= {j, k}, then
A X B = {(r,j), (w,j), (b,j), (r,k), (w,k), (b.k)}

6. Section 2.2. Historical numeration systems: Egyptian, Babylonian, Roman and Mayan.

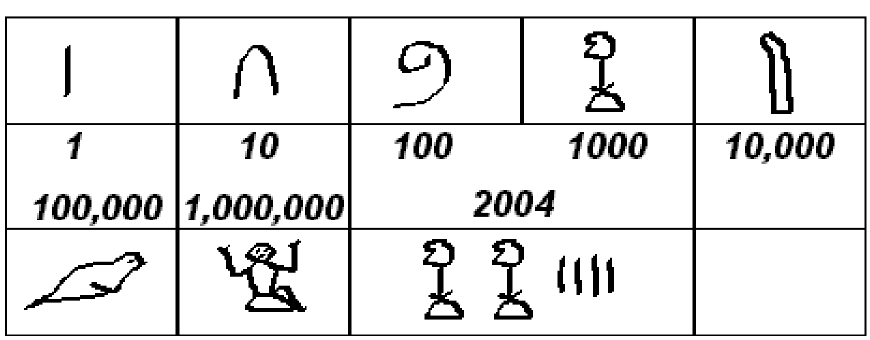
Maya Mathematics

Instead of ten digits like we have today, the Maya used a base number of 20. (Base 20 is vigesimal.) They also used a system of bar and dot as "shorthand" for counting. A dot stood for one and a bar stood for five. In the following table, you can see how this works.



Egyptian Numbers & Calendars

The Egyptians used, and possibly invented, the base ten number system. In early times, pre-dating the Old Kingdom around 2500BC, they used a basic system of hieroglyphic numerals where 1,10,100 etc were represented as single symbols:



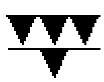
Roman Numerals

One	I	Eleven	XI	Thirty	XXX
Two	II	Twelve	XII	Forty	XL
Three	III	Thirteen	XIII	Fifty	L
Four	IV	Fourteen	XIV	Sixty	LX
Five	V	Fifteen	XV	Seventy	LXX
Six	VI	Sixteen	XVI	Eighty	LXXX
Seven	VII	Seventeen	XVII	Ninety	XC
Eight	VIII	Eighteen	XVIII	One hundred	C
Nine	IX	Nineteen	XIX	Five hundred	D
Ten	X	Twenty	XX	One thousand	M

I = 1
V = 5
X =10
L = 50
C =100
D = 500
M =1000

The Babylonian Number System

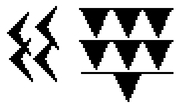
A base 60 system (sexigesimal). The Babylonians had a very advanced number system even for today's standards. Base ten is what we use today. Rather than a base ten (decimal) system the Babylonians divided the day into twenty-four hours, each hour into sixty minutes, and each minute to sixty seconds. This form of counting has survived for four thousand years. Any number less than 10 had a wedge that pointed down.

Example: 4 

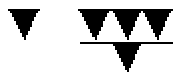
The number 10 was symbolized by a wedge pointing to the left.

Example: 20 

Numbers less than 60 were made by combining the symbols of 1and 10.

Example: 47 

As with our numbering system, the Babylonian numbering system utilized units, ie tens, hundreds, thousands.

Example: 64 

However, they did not have a symbol for zero, but they did use the idea of zero. When they wanted to express zero, they just left a blank space in the number they were writing.

When they wrote "60", they would put a single wedge mark in the second place of the numeral.

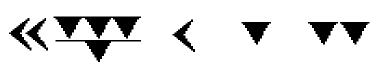
When they wrote "120", they would put two wedge marks in the second place.

Following are some examples of larger numbers.

Example: 79883

$(22*60^2) + (11*60) + 23$

Example: 5220062



$(24*60^3) + (10*60^2) + (1*60) + 2$

7. **Section 2.3**. Important features and advantages of our Hindu-Arabic system, other number bases, counting and conversions. Two important features are:

> *1. Digits, **0, 1, 2, 3, 4, 5, 6, 7, 8, 9:** These **ten** symbols or digits can be used in any possible combination to represent all possible numbers.*
>
> *2. Grouping by tens **(decimal system)** Grouping things into sets of 10 is a basic principle of the Hindu-Arabic system, most likely because people have 10 "digits" on our two hands. (the word digit literally means "finger" or "toe)*

8. **Section 2.4**. Understanding functions, including ways to represent them (see pictures in text), domain and range, arithmetic and geometric sequences. (see # 4 above.) Functions are specific types of relations:

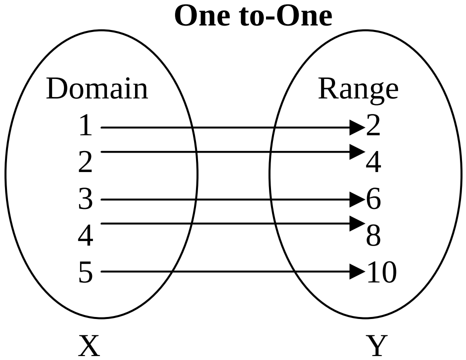
1. A ***FUNCTION*** is a relation the matches each element of the first set to an element of a second set in such a way that no element in the first set is assigned to two elements in the second set.

2. Terms arranged in order where the ***first term*** is called the **initial term.** For example**;** The sequence of consecutive even counting numbers listed in *increasing order* is 2, 4, 6, 8, 10,…. Using Arrows is another way to show the sequence:

2, 4, 6, 8, 10,….

1 ➔ **2**, 2 ➔ **4**, 3 ➔ **6,** 4 ➔ **8**, 5➔ **10,**…

One to-One

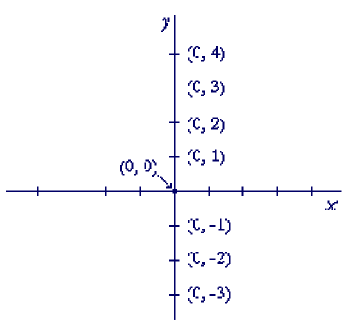


Set A = {1, 2,} Set B = {3, 4, 5)
A x B = 6 ordered pairs.

Set{ (**a**, b), (**c**, b),(**d**, b), (**e**, a)} ***Is a function***
Domain: All x values are different

Set{ (**a**, b), (c, b),(d, b), (**a**, a)} ***Not a function***
Domain: Two x values are the same

Cartesian Plane



Vocabulary/ Notation

Chapter 2

Number: A number is an IDEA, or ABSTRATION that represents a quantity

Numeral: A numeral is a SYMBOL that represents a number

Cardinal number: The CARDINAL NUMBER refers to the number of elements in a set

Ordinal number: The ORDINAL NUMBERS are concerned with order or numerical standing in.

Identification numbers; are the number that locate and identify us, Addresses, Telephone numbers, Social Security Numbers, PIN Numbers, Checking Account Number, Savings Account Number, etc.,

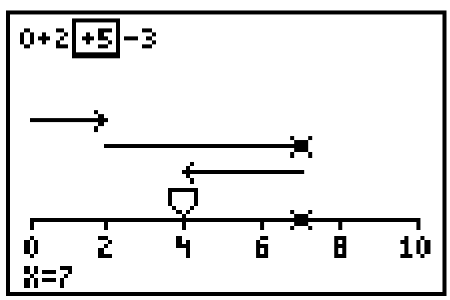
Number of sets: A method of ordering whole numbers. ***Let $a = n(A)$ and $b = n(B)$.***
Then $a < b$ or $b > a$, if A is equivalent to a proper subset of B.

Less than (<) , Greater than (>) Self explanatory. Example: $5 < 7$

Less than or equal to (≤) Self explanatory. Example: $5 \leq 7$ and also $7 \leq \mathbf{7}$

Greater than or equal to (≥) Self explanatory

Whole-number line: is a sequence of equally spaced marks where the numbers are represented by the marks beginning on the left at Zero.



Tally numeration system is composed of single strokes on for each object being counted. ||||| ||||| |||||
The advantage is its simplicity. Disadvantages are two: 1) Large number require many individual symbols and 2) it is difficult to read numerals for large numbers.

Grouping: improved the tally system by every 5 mark placed across the previous four.

卌 卌 卌 卌 || = 22

Egyptian numeration system is an additive system since the values for the various individual numerals are added together. There is no place value since the order of the symbols does not matter.

Additive numeration system is a system of mathematics where the values for the various individual numerals are added together.

Roman numeration system additive with a **subtractive feature,** developed by the Romans between 500 B.C. and A.D. 100 **. C C L X X X I** = 281 C=100, CL=150, X =10
Originally, 4 was written IIII. But later was written IV with I subtracted from V.
Originally, 9 was written VIIII. But later was written IX with I subtracted from X
Subtractive numeration is a system of mathematics where the values for the various individual numerals are subtracted from each other for specific values.

1	**I**
5	**V**
10	**X**
50	**L**
100	**C**
500	**D**
1000	**M**

Positional numeration system: The Roman system is a positional system since the position of a numeral can effect the value of the number being represented. Any time a lesser numeral is placed in front of a larger numeral, subtract for the results.

MC = 1100 but CM = 900; LX = 60 but XL = 40.

Multiplicative numeration system: The Roman system had multiplicative features. Placing a horizontal bar above any numeral represented multiplication.

$\overline{\text{V}}$ meant 5 x 1000, or 5000; **$\overline{\text{XI}}$ meant 11 x 1000, or 11,000**

Babylonian numeration system is a base 60 system with place values of 60^2, 60, and 1

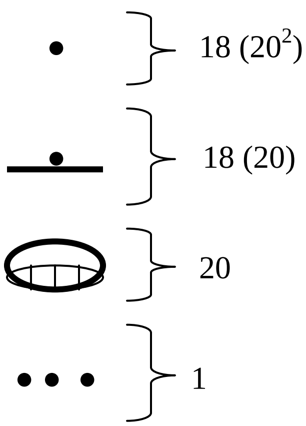
Place-value system is how the Babylonians represented different values depending on the place in which they were written. 60^2 place then the 60 place then the 1 place.
See examples above on pages 6-7.

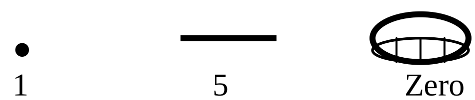
Place holder was represented by a vacant space but still caused confusion.

 Two tens written next to each other could be read as 20 or 10(60) + 10 = 610, or $(10*60^2) + (10*60) = 3660$.

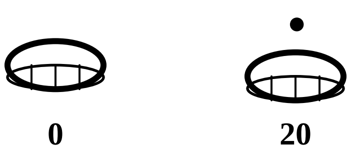
Although their place holder acts much like our zero, the Babylonians did not recognize zero as a number.

Mayan numeration system is a system of mathematics developed between A.D. 300 and A.D. 900 where the values for the various individual numerals are added together. This numeration system was a vertical place-value system that used only three elementary numerals that introduced a symbol for zero. It began with place values based on powers of 20. A later version of this system used an 18 for one of the 20s in each place value from 20^2 and above. Thus 20^2 became 18 * 20 as shown below.





Zero



Review using your class notes, quizzes, and the vocabulary lists in the textbook chapter reviews.

Do problems in the Chapter reviews and take the chapter tests as though they are practice tests. Check your answers with those in the back of the textbook.