MAT 111, section ______ ________ **Name** Answers

Test 3, version A, Fall 2009 **100 points** **Seat: Letter & Number** ________

Directions: Show all work algebraically unless otherwise indicated. If solving graphically, label the graph and the solution on the graph. If rounding is necessary, round to the **nearest thousandth**.

20 points [Chapters 5 and 6 sections 1-4.]

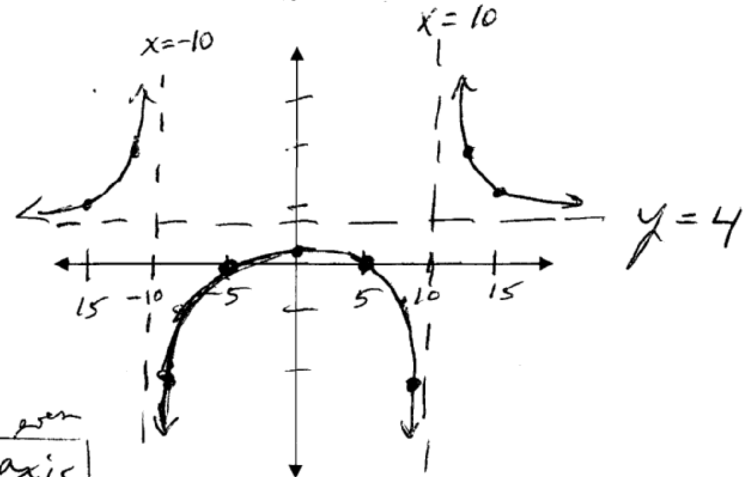
5.2-5.3

1. For the rational function $y = \frac{4x^2 - 100}{x^2 - 100}$ find the following, show your work:

-3pts if just center pts is shown with no asymptotes

2pts a. Domain $\boxed{\text{All real \#s except } x = \pm 10}$

6pts g. Graph, including the information in a-e; plot additional points as needed.



2pts b. y-intercept when $x=0$ $\boxed{(0,1)}$ $y = \frac{4(0^2)-100}{(0^2)-100} = 1$

2pts c. x-intercept(s)
$0 = 4x^2 - 100$
$25 = x^2$
$\pm 5 = x$
$\boxed{(5,0) \quad (-5,0)}$

2pts d. vertical asymptote(s) undefined where den = 0 $\boxed{x = 10 \quad x = -10}$

2pts e. end behavior (horizontal asymptote) powers same ratio of coefficients $\boxed{y = 4}$

1 if y=1

2pts f. symmetry? even $f(-x) = f(x)$ $\boxed{\text{sym w.r.t. } y\text{-axis}}$

2pts h. Is this function one-to-one? No Why or why not? Does not pass the horizontal line test, or 2 different x-values give the same y-values.

8 points

2. Find the following for the rational functions below:

3 pts a. The horizontal asymptote of $f(x) = \frac{3x-4}{x^2+2}$. $\boxed{y = 0}$ since the degree of numerator < deg of den.

5 pts b. The x and y coordinates of the hole in the graph of $y = \frac{x^2-9}{x^2-2x-3}$. $= \frac{(x-3)(x+3)}{(x-3)(x+1)}$

at $x = 3$ y would have been $\frac{3}{2}$ $\boxed{\left(3, \frac{3}{2}\right)}$

$x = -1$ vert asy.
$x = 3$ "hole"

12 points

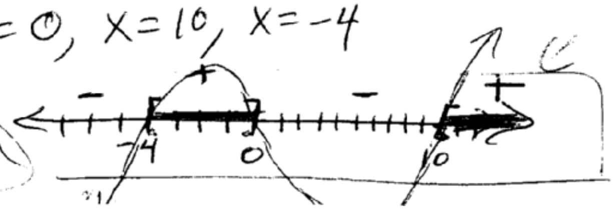
3. Solve the inequality: $x^3 - 6x^2 \geq 40x$. Draw your solution on a number line and write it in interval notation.

5pts
$x^3 - 6x^2 - 40x \geq 0$
$x(x^2 - 6x - 40) \geq 0$
$x(x-10)(x+4) \geq 0$
$x = 0, \ x = 10, \ x = -4$

optional graph to see where ≥ 0

4pts $\boxed{[-4, 0] \cup [10, \infty)}$

check values in each interval



3pts

40

10 points

4. For $f(x) = \frac{5}{x-4}$ and $g(x) = x^2 - 5$ find the following:

1pt a. Domain of f all real #s ~~except~~ $x \neq 4$

2pts d. Domain of $f \circ g$ all real #s except $x = \pm 3$

1pt b. Domain of g all real #s

e. $f \circ g(7)$ (simplify completely)

3pts $(f \circ g)(7) = \frac{5}{7^2 - 9} = \frac{5}{49-9} = \frac{5}{40} = \frac{1}{8} = .1$

c. $f \circ g(x)$ (simplify completely)

3pts $f(g(x)) = \frac{5}{(x^2-5)-4} = \frac{5}{x^2-9}$

or $g(7) = 7^2 - 5 = 44$

$f(44) = \frac{5}{40-4} = \frac{5}{40} = \frac{1}{8}$

5 points

5. If $(f \circ g)(x) = \sqrt[3]{x^2 - 6x + 5}$, find $f(x)$ and $g(x)$. Then 2pts $f(x) = \sqrt[3]{x}$ and 3pts $g(x) = (x^2 - 6x + 5)$

ok if $\sqrt[3]{x+5}$ + $(x^2 - 6x)$

20 points

6. . Graph $f(x) = 3^{(x)} - 1$; using a table of values, list at least four points. switch coordinates 2pts

2pts Domain of f(x): all real #s

2pts Range of f(x): all real #s > −1

2pts Asymptote: $y = -1$

2pts

x	$f(x)$
0	$3^0 - 1 = 0$
1	$3 - 1 = 2$
2	$9 - 1 = 8$
−1	$\frac{1}{3} - 1 = -\frac{2}{3}$

x	$f^{-1}(x)$
0	0
2	1
8	2
$-\frac{2}{3}$	−1

$f(x) = 3^x - 1$

$y = 3^x - 1$

Graph $f^{-1}(x)$ on the same axes, using a table of values, include at least four points and any asymptotes.
Determine the equation of $f^{-1}(x)$ $f^{-1}(x) = \log_3(x+1)$ 3pts (Show your work below.)

orig func $f(x) = 3^x - 1$

$y = 3^x - 1$

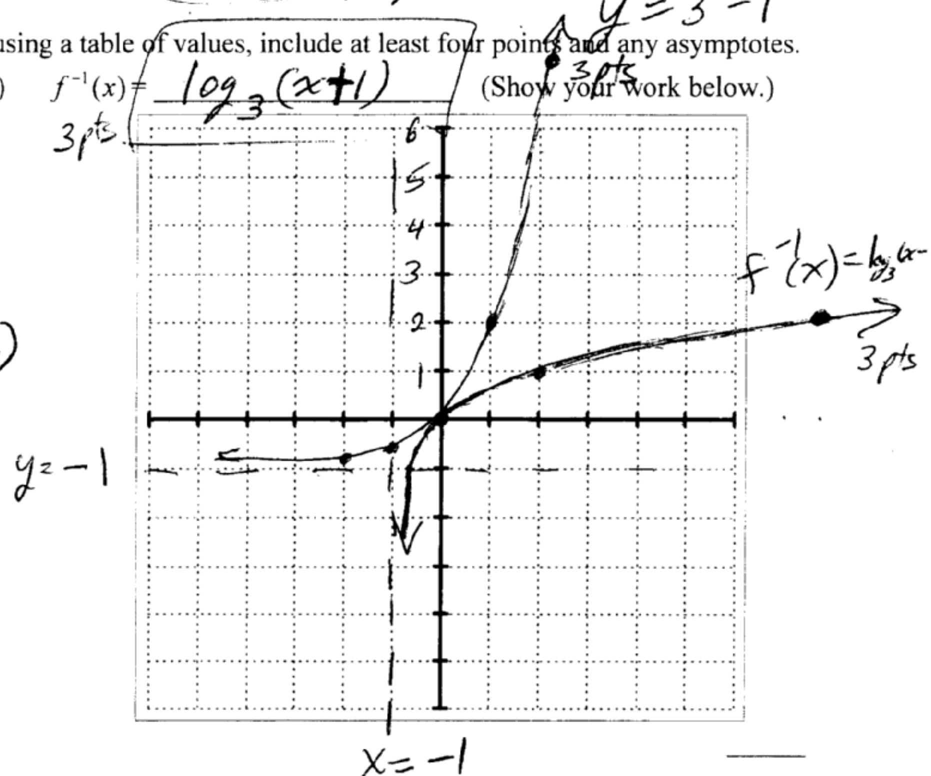
(switch x & y then solve for y)

$x = 3^y + 1$

$(x+1) = 3^y$

$y = \log_3(x+1)$

$f^{-1}(x) = \log_3(x+1)$



9 points (3 each)

7. Evaluate the following by rewriting in exponential form and simplifying. Show your work:

a. $\log_2\left(\frac{1}{16}\right) = x$

$2^x = \frac{1}{16}$

$2^x = 2^{-4}$

$\boxed{x = -4}$

b. $\log_5(125) = x$

$5^x = 125$

$5^x = 5^3$

$\boxed{x = 3}$

c. $\log_3(1) = x$

$3^x = 1$

$\boxed{x = 0}$

16 points (4 each)

8. Solve for x. Give exact answers and where appropriate, approximate answers rounded to 3 decimal places.

a. $5^{4x+6} = 25^x$

$5^{4x+6} = 5^{2x}$

$4x+6 = 2x$

$2x = -6$

$x = -3$

b. $8^{x-1} = \frac{1}{4}$

$8^{x-1} = \frac{1}{2^2}$

$(2^3)^{x-1} = 2^{-2}$

$2^{3x-3} = 2^{-2}$

$3x-3 = -2$

$3x = 1$

$\boxed{x = 1/3 \approx .333}$

c. $e^{2x} = 7$

$\log_e 7 = 2x$

or $\ln 7 = 2x$

$x = \frac{\ln 7}{2}$

$x \approx \frac{1.945910\ldots}{2}$

$\boxed{x \approx 0.973}$

d. $\log_3(2x-1) = 4$

$3^4 = 2x-1$

$81 = 2x-1$

$82 = 2x$

$\boxed{41 = x}$

25