

**MAT 321-1**  
**HOMEWORK ASSIGNMENT #3**  
**HAND IN THURSDAY, FEBRUARY 9**

For each problem, show your work (by hand, except for #6) neatly on a separate sheet of paper.

1. Find   a)  $1000 \pmod{98}$   
          b)  $-100 \pmod{12}$
2. Let  $m$  be a positive integer.
  - a) List all possibilities for  $m$  if  $20 \equiv 4 \pmod{m}$ .
  - b) List all possibilities for  $m$  if  $30 \equiv 13 \pmod{m}$ .
3. Prove directly using the equivalent equation form (i.e. without using Theorem 4.3, 4.5 or 4.7) that if  $a, b, c, m \in \mathbf{Z}$  and  $m > 0$ , then

$$a \equiv b \pmod{m} \quad \Rightarrow \quad a^2 \equiv b^2 \pmod{m}$$

4. Use properties of congruences to solve, step-by-step, for  $x$ , given that  $x$  is an integer between 1 and 20:                    $9x - 14 \equiv 70 \pmod{60}$   
[HINT: Use the Cancellation Law at some point in your solution.]
5. Use the modular exponentiation algorithm to find  $37^{101} \pmod{87}$   
(Show all the steps. You may use a calculator or Maple to help with the individual steps.)
6. Use Maple's modular exponentiation command    $> b\&^N \pmod{m}$ ;  
to find:
  - a)  $13^{15000} \pmod{27397}$
  - b)  $13^{27396} \pmod{27397}$
7. a) Find a complete system of residues modulo 9 consisting entirely of odd integers.  
b) Show why it is not possible to find a complete system of residues modulo 10 consisting entirely of odd integers.
8. The routing number at the bottom of checks issued by a bank is a nine digit number. The first four digits are the Federal Reserve number, the next four digits are the individual bank identifier, and the ninth digit is a check digit. If the digits are named (left to right)  $x_1, x_2, \dots, x_8, C$ , then the checksum rule for bank routing numbers is:  
$$3(x_1 + x_4 + x_7) + 7(x_2 + x_5 + x_8) + (x_3 + x_6 + C) \equiv 0 \pmod{10}$$
  - a) Use properties of congruences to show that the routing number checksum rule detects all single errors.
  - b) Does the routing checksum rule detect all transposition errors? Justify your answer.