

MAT 321-1
HOMEWORK ASG. #2
HAND IN THURSDAY, FEBRUARY 2

For each problem, show your work neatly on a separate sheet of paper.

1. Find $\gcd(646, 1406)$ by our efficient method (the method of Theorem 3.7).
2. Find $\gcd(45, 90, 84, 60)$ by the "pairing method" (the method of Lemma 3.2).
3. For each set of integers, find the gcd and write the gcd as a linear combination of the given integers.
 - a) 104, 39
 - b) 216, 145
 - c) 21, 33, 77
4. Find all ways to write 3 as a linear combination of 15 and 51.
5. Suppose n is an arbitrary positive integer. Show that $(2n+15, 3n+20) = (n, 5)$.
6. Let n be a positive integer. Is it always true that at least one of the four integers

$$n, \quad n+2, \quad n+4, \quad n+6$$

is divisible by 4? Justify your answer.

7. Let m be a positive integer. Is it always true that at least one of the five integers

$$m, \quad m+3, \quad m+6, \quad m+9, \quad m+12$$

is divisible by 5? Justify your answer.

[HINT: For problems 6 and 7, if you think an answer is NO, all you have to do for a reason is produce a counterexample. If you think an answer is YES, you need to give a general case-by-case argument based on a classification of integers according to the Division Algorithm, along the lines of the in-class example on Jan. 24 showing that at least one of $p, p+2, p+4$ is always divisible by 3.]