Section 7.2

(3) $\int 4(2x+3)^4\,dx$

Let $u = 2x+3$ or $2x+3 = u$

$\frac{du}{dx} = \frac{d}{dx}(2x+3)$

$\frac{du}{dx} = 2 \Rightarrow du = 2\,dx$

$dx = \frac{du}{2}$

$= \int 4(u)^4 \frac{du}{2}$

$= \int 2u^4\,du$

$= \frac{2(u^5)}{5} + C$

$= \frac{2(2x+3)^5}{5} + C$

(5) $\int \frac{2\,dm}{(2m+1)^3}$

$\int 2(2m+1)^{-3}\,dm$

Let $u = 2m+1$

$\frac{du}{dm} = \frac{d}{dm}(2m+1)$

$\frac{du}{dm} = 2$

$\frac{du}{2} = dm$

$\int 2(u)^{-3} \frac{du}{2}$

$\int u^{-3}\,du$

$= \frac{u^{-2}}{-2} + C$

$= -\frac{1}{2}(2m+1)^{-2} + C$

$= \frac{-(2m+1)^{-2}}{2} + C$

(7) $\int \frac{2x+2}{(x^2+2x-4)^4}\,dx$

$\int (2x+2)(x^2+2x-4)^{-4}\,dx$

Let $u = x^2+2x-4$

$\frac{du}{dx} = \frac{d}{dx}(x^2+2x-4)$

$\frac{du}{dx} = 2x+2$

$\frac{du}{2x+2} = dx$

$\int (2x+2)(u^{-4}) \frac{du}{2x+2}$

$\int u^{-4}\,du$

$= \frac{u^{-3}}{-3} + C$

$= -\frac{1}{3}(x^2+2x-4)^{-3} + C$

$= \frac{-(x^2+2x-4)^{-3}}{3} + C$

(9) $\int z\sqrt{z^2-5}\,dz$

$\int z(z^2-5)^{1/2}\,dz$

Let $u = z^2-5$

$\frac{du}{dz} = \frac{d}{dz}(z^2-5)$

$\frac{du}{dz} = 2z$

$\frac{du}{2z} = dz$

$\int z u^{1/2} \frac{du}{2z}$

$\int \frac{u^{1/2}}{2}\,du$

$= \frac{1}{2}\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{2}\left(\frac{2}{3}\right)u^{3/2} + C$

$= \frac{1}{3}(z^2-5)^{3/2} + C$

$= \frac{(z^2-5)^{3/2}}{3} + C$

(11) $\int -4e^{2p}\,dp$

Let $u = 2p$

$\frac{du}{dp} = \frac{d}{dp}(2p) = 2$

$\frac{du}{2} = dp$

$\int -4e^u \frac{du}{2}$

$-2\int e^u\,du$

$= -2e^u + C$

$= -2e^{2p} + C$

(13) $\int 3x^2 e^{2x^3}\,dx$

Let $u = 2x^3$

$\frac{du}{dx} = \frac{d}{dx}(2x^3) = 6x^2$

$\frac{du}{6x^2} = dx$

$\int 3x^2 e^u \frac{du}{6x^2}$

$\int \frac{3x^2}{6x^2} e^u\,du$

$\int \frac{1}{2} e^u\,du$

$= \frac{1}{2}e^u + C$

$= \frac{1}{2}(e^u) + C$

$= \frac{e^{2x^3}}{2} + C$

(15) $\int (1-t)e^{2t-t^2}\,dt$

Let $u = 2t - t^2$

$\frac{du}{dt} = \frac{d}{dt}(2t-t^2) = 2-2t$

$\frac{du}{dt} = 2(1-t) \qquad \frac{du}{2(1-t)} = dt$

$\int (1-t)e^{u}\frac{du}{2(1-t)}$

$\frac{1}{2}\int e^{u}\,du$

$= \frac{1}{2}e^{2t-t^2} + c$

$= \frac{e^{2t-t^2}}{2} + C$

(17) $\int \frac{e^{1/z}}{z^2}\,dz$

$\int e^{z^{-1}} z^{-2}\,dz$

Let $u = z^{-1}$

$\frac{du}{dz} = \frac{d}{dz}(z^{-1}) = -z^{-2}$

$\frac{du}{-z^{-2}} = dz$

$\int e^{u} z^{-2}\frac{du}{-z^{-2}}$

$= \int -e^{u}\,du$

$-e^{u} + c$

$= -e^{z^{-1}} + c$

$= -e^{1/z} + c$

(19) $\int (x^3+2x)(x^4+4x^2+7)^8\,dx$

Let $u = x^4+4x^2+7$

$\frac{du}{4x^3+8x} = dx$

$\frac{du}{4(x^3+2x)} = dx$

$\int (x^3+2x)(u)^8\frac{du}{4(x^3+2x)}$

$\int \frac{1}{4}u^8\,du$

$= \frac{1}{4}\frac{u^9}{9} + C$

$= \frac{1}{36}(x^4+4x^2+7)^9 + c$

$= \frac{(x^4+4x^2+7)^9}{36} + C$

(21) $\int \frac{2x+1}{(x^2+x)^3}\,dx$

Let $u = x^2+x$

$\frac{du}{2x+1} = dx$

$\int (2x+1)u^{-3}\frac{du}{2x+1}$

$\int u^{-3}\,du$

$= \frac{u^{-2}}{-2} + c$

$= \frac{-1}{2(x^2+x)^2} + C$

p.375 * ← Have extra steps involved. → *

(23) $\int p(p+1)^5\,dp$

Let $u = p+1 \rightarrow p = u-1$

$\frac{du}{dp} = \frac{d}{dp}(p+1) = 1 \longrightarrow du = dp$

$\int (u-1)(u)^5\,du = \int (u^6-u^5)\,du$

$= \frac{u^7}{7} - \frac{u^6}{6} + C$

$= \frac{1}{7}(p+1)^7 - \frac{1}{6}(p+1)^6 + C$

$= \frac{(p+1)^7}{7} - \frac{(p+1)^6}{6} + c$

(25) $\int \frac{u}{\sqrt{u-1}}\,du$

$\int u(u-1)^{-1/2}\,du$

Let $s = u-1 \qquad s-1 = u$

$\frac{ds}{du} = \frac{d}{du}(u-1) = 1 \qquad ds = du$

$\int (s-1)s^{-1/2}\,ds$

$\int (s^{1/2} - s^{-1/2})\,ds$

$\frac{s^{3/2}}{3/2} - \frac{s^{1/2}}{1/2} + c$

$= \frac{2}{3}s^{3/2} \cdot \frac{1}{2}s^{1/2} + c$

$= \frac{2}{3}(u-1)^{3/2} - \frac{1}{2}(u-1)^{1/2} + c$

$= \frac{2(u-1)^{3/2}}{3} - \frac{(u-1)^{1/2}}{2} + C$

(27) $\int (\sqrt{x^2+12x})(x+6)\,dx$

$\int (x^2+12x)^{1/2}(x+6)\,dx$

Let $u = x^2+12x \qquad \frac{du}{dx} = \frac{d}{dx}(x^2+12x)$

$\frac{du}{dx} = 2x+12 = 2(x+6)$

$\frac{du}{2(x+6)} = dx$

$\int u^{1/2}(x+6)\frac{du}{2(x+6)}$

$\frac{1}{2}\int u^{1/2}\,du$

$(\frac{1}{2})(\frac{2}{3})u^{3/2} + c$

$\frac{1}{3}(x^2+12x)^{3/2} + C$

$= \frac{(x^2+12x)^{3/2}}{3} + C$

(29) $\int \left(\frac{t}{t^2+2}\right)dt$

Let $u = t^2+2$

$\frac{du}{dt} = \frac{d}{dt}(t^2+2) = 2t$

$\frac{du}{2t} = dt$

$\int \frac{t}{u}\frac{du}{2t}$

$\frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln(t^2+2) + C$

$= \frac{\ln(t^2+2)}{2} + c$

(31) $\int \frac{(1+\ln x)^2}{x}\,dx$

$\int (\frac{1}{x})(1+\ln x)^2\,dx$

Let $u = 1+\ln x$

$\frac{du}{dx} = \frac{d}{dx}(1+\ln x) = \frac{1}{x}$

$x\,du = dx$

$\int (\frac{1}{x})(u)^2\, x\,du$

$\int u^2\,du$

$u^3/3 + C$

$\frac{1}{3}(1+\ln|x|)^3 + C$

(33) $\int \frac{e^{2x}}{e^{2x}+5}\,dx$

Let $u = e^{2x}+5$

$\frac{du}{dx} = \frac{d}{dx}(e^{2x}+5) = 2e^{2x}$

$\frac{du}{2e^{2x}} = dx$

$\int \frac{e^{2x}}{u}\ \frac{du}{2e^{2x}}$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}(\ln(e^{2x}+5)) + C$

(37) $R'(x) = 2x(x^2+50)^2$

$R(x) = \int 2x(x^2+50)^2\,dx$

Let $u = x^2+50$

$\frac{du}{dx} = \frac{d}{dx}(x^2+50) = 2x$

$\frac{du}{2x} = dx$

$R(x) = \int 2x(u^2)\frac{du}{2x}$

$R(x) = $ " $\int u^2\,du$ "

$R(x) = $ " $\frac{1}{3}u^3 + C$ "

$R(x) = \frac{1}{3}(x^2+50)^3 + C$

$R(3) = \frac{1}{3}(3^2+50)^3 + C = 206,379$

$\frac{1}{3}(59)^3 + C = 206,379$

$C = 137,919.33$

a) $R(x) = \frac{1}{3}(x^2+50)^3 + 137,919.33$

b) $R(x) = \frac{1}{3}(x^2+50)^3 + 137,919.33 = 450,000$

$\frac{1}{3}(x^2+50)^3 + 137,919.33 = 450,000$

$\frac{1}{3}(x^2+50)^3 = 312080.67$

$(x^2+50)^3 = 936242.01$

$(x^2+50) \approx 98$

$x^2 \approx 48$

$x \approx 7$ b/c (negatives values are extraneous)