positive slope → increasing
negative slope → decreasing

Section 5.4 [p. 299; 3, 7, 9, 15, 25, OPTIONAL 35 - 39 (odd)]

Ⓐ Domain Ⓑ Critical points Ⓒ Intervals inc./dec. Ⓓ Point of Inflection
Ⓔ Intervals concave up/down Ⓕ Intercepts Ⓖ Asymptotes

③ $f(x) = -2x^3 - 9x^2 + 108x - 10$

Ⓐ Domain : $(-\infty, \infty)$

$f'(x) = -6x^2 - 18x + 108$

$-6(x^2 + 3x - 18) = 0$

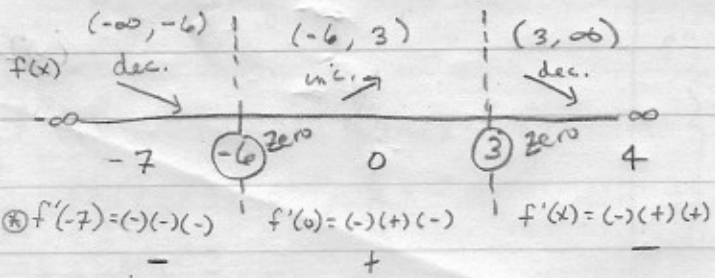
④ $-6(x+6)(x-3) = 0$

$x = -6 \quad x = 3$: Critical numbers

Ⓑ Critical points : $(-6, -550)$ & $(3, 179)$

$f(-6) = -2(-6)^3 - 9(-6)^2 + 108(-6) - 10 = -550$

$f(3) = -2(3)^3 - 9(3)^2 + 108(3) - 10 = 179$

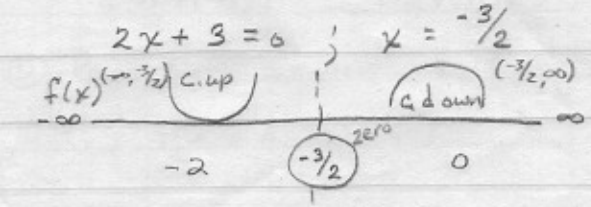


Ⓒ Increasing : $(-6, 3)$

Decreasing : $(-\infty, -6)$ and $(3, \infty)$

④ $f''(x) = -12x - 18 = -6(2x+3)$

$2x + 3 = 0 \quad ; \quad x = -3/2$



④ $f''(-2) = (-)(-)$ $\quad f''(0) = (-)(+)$

$+ \qquad -$

→ point of inflection occurs at $x = -3/2$

Ⓓ Point of Inflection : $(-3/2, -185.5)$

$f(-3/2) = -2(-3/2)^3 - 9(-3/2)^2 + 108(-3/2) - 10 = -185.5$

Ⓔ Concave up : $(-\infty, -3/2)$

Concave down : $(-3/2, \infty)$

Ⓕ y-intercepts $(x = 0)$: -10

$f(0) = -10$

x-intercepts $(f(x) = 0$: Use Calc.

$x \approx$

Ⓖ None

Interval :	$(-\infty, -6)$	$(-6, -3/2)$	$(-3/2, 3)$	$(3, \infty)$
Sign of $f'(x)$	−	+	+	−
Sign of $f''(x)$	+	+	−	−
Inc./Dec.	↘	↗	↗	↘
Concave up/down	∪	∪	∩	∩
SHAPE of GRAPH $f(x)$	↓	↓	↓	
	$(-6, 550)$	$(-3/2, -185.5)$	$(3, 179)$	

⑦ $f(x) = x^4 - 18x^2 + 5$

Ⓐ Domain : $(-\infty, \infty)$

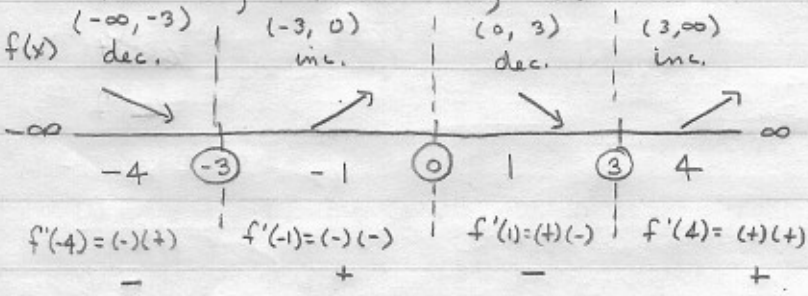
$f'(x) = 4x^3 - 36x \overset{*}{=} 4x(x^2 - 9)$

$4x(x^2 - 3) = 0$

$x = 0,\ x = -3,\ x = 3$: Critical #'s

Ⓑ Critical Points : $(0, 5)$, $(-3, -76)$, $(3, -76)$

$f(0) = 5$; $f(-3) = -76$; $f(3) = -76$



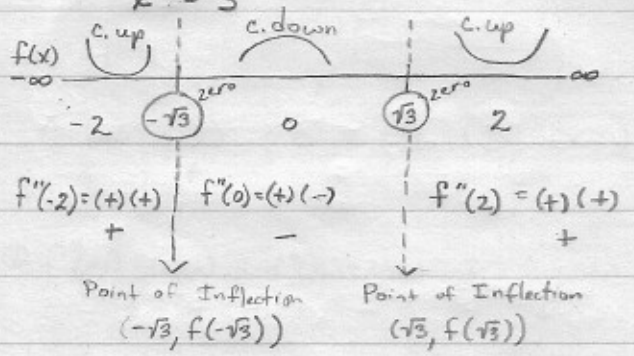
Ⓒ Increasing : $(-3, 0)$ and $(3, \infty)$

Decreasing : $(-\infty, -3)$ and $(0, 3)$

$f''(x) = 12x^2 - 36 \overset{*}{=} 12(x^2 - 3)$

$12(x^2 - 3) = 0$; $x = \pm\sqrt{3}$

$x^2 = 3$



Ⓓ Points of Inflection :

$(\sqrt{3}, -40)$ and $(\sqrt{3}, -40)$

Ⓔ Concave up : $(-\infty, -\sqrt{3})$ and $(\sqrt{3}, \infty)$

Concave down : $(-\sqrt{3}, \sqrt{3})$

Ⓕ x-intercepts :

$x^4 - 18x^2 + 5 = 0$ "Quadratic Form" $au^2 + bu + c$

$x^2 = \dfrac{18 \pm \sqrt{(-18)^2 - 4(1)(5)}}{2}$

	$(-\infty, -3)$	$(-3, -\sqrt{3})$	$(-\sqrt{3}, 0)$	$(0, \sqrt{3})$	$(\sqrt{3}, 3)$	$(3, \infty)$
f'	−	+	+	−	−	+
f''	+	+	−	−	+	+
	↘	↗	↗	↘	↘	↗
	∪	∪	∩	∩	∪	∪
$f(x)$		↓ $(-3, -76)$	↓ $(-\sqrt{3}, -40)$	↓ $(0, 5)$	↓ $(\sqrt{3}, -40)$	↓ $(3, -76)$

(25) $y = xe^{-x}$

$y' = x(-e^{-x}) + (e^{-x})$

$= -e^{-x}(x-1)$

Domain: $(-\infty, \infty)$

Zero: $-e^{-x}$ will never equal to 0.

$(x-1) = 0 \quad x = 1$

(B) Critical Point: $(1, f(1)) = (1, e^{-1}) \approx (1, .368)$

(C) Inc. $(-\infty, 1)$ Dec. $(1, \infty)$

0 2

$f'(0) = -e^{(-1)}(0-1) = -e^{-1}(-1) = e^{-1}$ ⊕↗

$f'(2) = -e^{-2}(2-1) = -e^{-2}(1) = -e^{-2}$ ⊖↘

$y'' = -e^{-x}(1) + (x-1)(e^{-x})$

$= e^{-x}(-1 + x - 1) = e^{-x}(x-2)$

Zero: e^{-x} will never equal to zero.

$(x-2) = 0 \quad x = 2$

(E) Concave down ∩ $(-\infty, 2)$ Concave up ∪ $(2, \infty)$

0 ⊖ 3 ⊕

$f''(0) = e^0(0-2) = (1)(-2) = -2$ ⊖∩

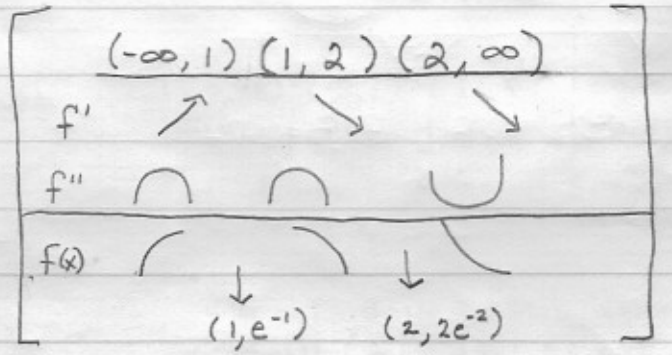
$f''(3) = e^{-3}(3-1) = e^{-3}(2) = 2e^{-3}$ ⊕∪

(D) Point of Inflection: $(2, f(2)) = (2, 2e^{-2}) \approx (2, .271)$

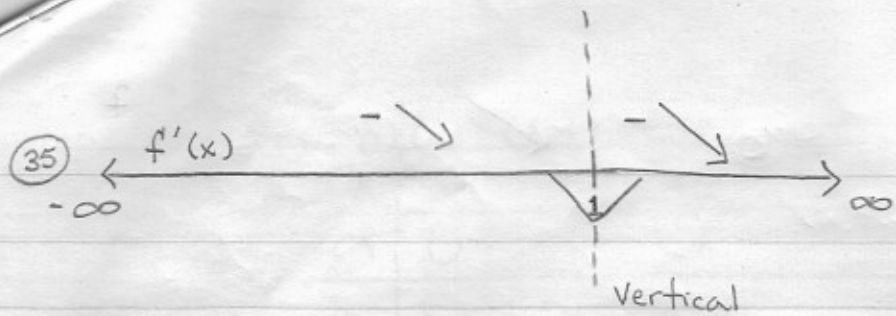
(F) y-int: $y = 0$

x-int: $x = 0$

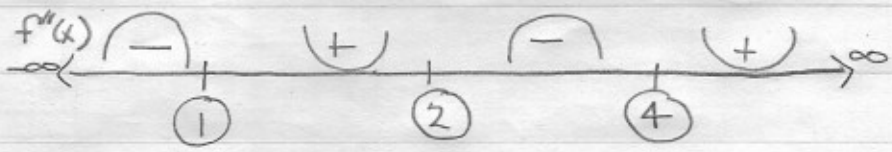
(G) None

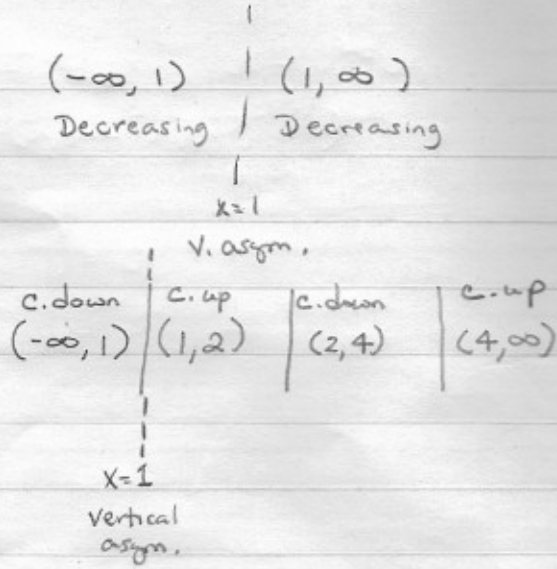


(35)



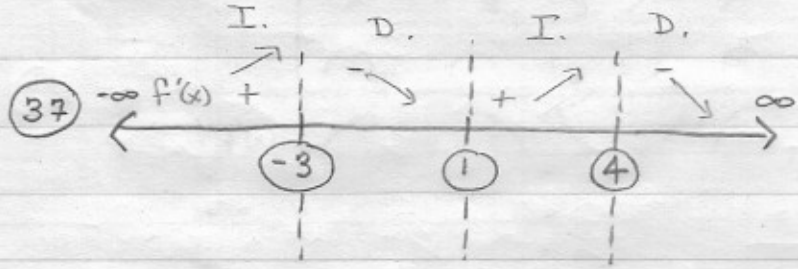
Vertical asymp.

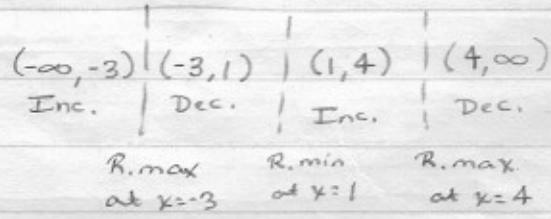


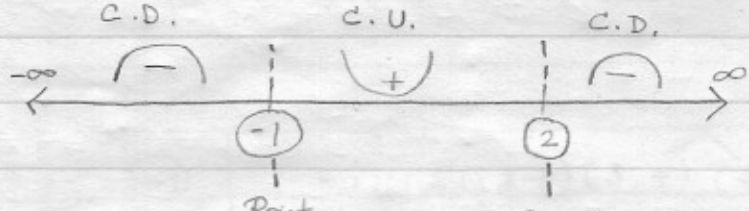


Horizontal Asym.: $y = 2$

(37)

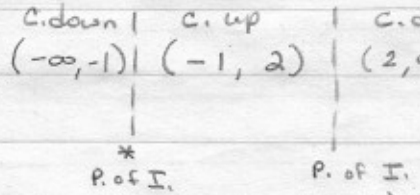




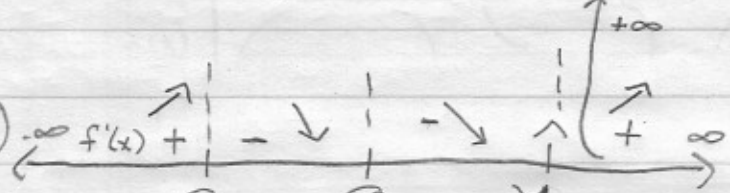


Point of Inflection $(-1, 3)$

Pt. of Inflection $(2, 4)$



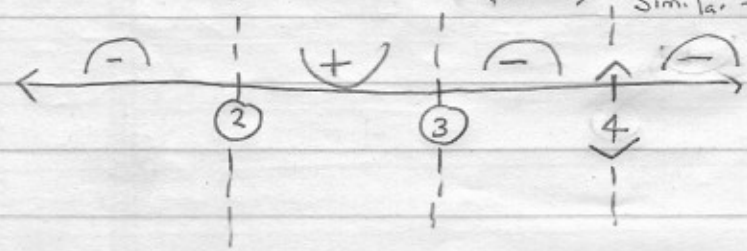
(39)



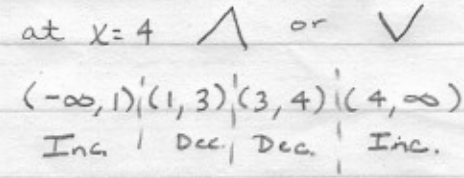
Not differentiable at $x = 4$

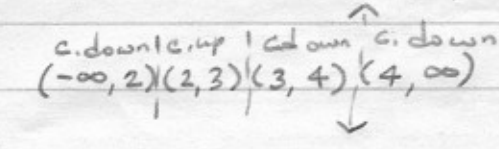
$(1, 5)$ (R. max)

$-\infty$ (R. min)



at $x = 4$ $\wedge$ or $\vee$





(35)

	$(-\infty, 1)$	$(1,2)$	$(2,4)$	$(4,\infty)$
$f'(x)$	↘	↘	↘	↘
$f''(x)$	∩	∪	∩	∪
$f(x)$	⌝	⌟	⌝	⌟

(37)

	$(-\infty,-3)$	$(-3,-1)$	$(-1,1)$	$(1,2)$	$(2,4)$	$(4,\infty)$
f'	↗	↘	↘	↗	↗	↘
f''	∩	∩	∪	∪	∩	∩
$f(x)$	⌜	⌝	⌟	⌞	⌜	⌝

(39)

	$(-\infty,1)$	$(1,2)$	$(2,3)$	$(3,4)$	$(4,\infty)$
f'	↗	↘	↘	↘	↗
f''	∩	∩	∪	∩	∩
$f(x)$	⌜	⌝	⌟	⌝	⌜