Section 4.5 p. 245 [1-39, 47, 49] odd

① $y = \ln(8x)$ $\begin{cases} g'(x) = 8 \\ g(x) = 8x \end{cases}$

$\begin{cases} y = \ln(g(x)) \\ \dfrac{dy}{dx} = \dfrac{g'(x)}{g(x)} \end{cases}$

$\dfrac{dy}{dx} = \dfrac{8}{8x} = \dfrac{1}{x}$

③ $y = \ln(3-x)$

$\dfrac{dy}{dx} = \dfrac{-1}{(3-x)}$

⑤ $y = \ln|2x^2 - 7x|$

$\dfrac{dy}{dx} = \dfrac{4x-7}{2x^2-7x}$

⑦ $y = \ln\sqrt{x+5}$

$= \ln(x+5)^{1/2} = \frac{1}{2}\ln(x+5)$

$\dfrac{dy}{dx} = \dfrac{1}{2}\left(\dfrac{1}{x+5}\right)$

$= \dfrac{1}{2(x+5)}$

⑨ $y = \ln(x^4 + 5x^2)^{3/2} = \frac{3}{2}\ln(x^4+5x^2)$

$\dfrac{dy}{dx} = \dfrac{3}{2}\left[\dfrac{4x^3+10x}{x^4+5x^2}\right]$

$= \dfrac{3}{2}\left[\dfrac{2x(2x^2+5)}{x^2(x^2+5)}\right]$

$= \dfrac{3(2x^2+5)}{x(x^2+5)}$

⑪ $y = \underbrace{-3x}_{u}\,\underbrace{\ln(x+2)}_{v}$

$\begin{cases} \dfrac{dy}{dx} = uv' + vu' \\ u' = -3 \qquad v' = \dfrac{1}{x+2} \end{cases}$

$\dfrac{dy}{dx} = (-3x)\left(\dfrac{1}{x+2}\right) + \ln(x+2)\,(-3)$

$= \dfrac{-3x}{x+2} - 3\ln(x+2)$

⑬ $S = \underbrace{t^2}_{u}\,\underbrace{\ln|t|}_{v}$

$\{ u' = 2t \qquad v' = \dfrac{1}{t}$

$\dfrac{ds}{dt} = t^2\left(\dfrac{1}{t}\right) + (2t)\ln|t|$

$= t + 2t\ln|t|$

⑮ $y = \dfrac{2\ln(x+3)}{x^2} \begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}$

$\begin{cases} \dfrac{dy}{dx} = \dfrac{vu' - uv'}{v^2} \\ u' = 2\left(\dfrac{1}{x+3}\right) \quad v' = 2x \end{cases}$

$\dfrac{dy}{dx} = \dfrac{x^2\left(\frac{2}{x+3}\right) - 2\ln(x+3)(2x)}{(x^2)^2}$

⑰ $y = \dfrac{\ln x}{4x+7} \begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}$

$\dfrac{dy}{dx} = \dfrac{(4x+7)(1/x) - (\ln x)(4)}{(4x+7)^2}$

⑲ $y = \dfrac{3x^2}{\ln x}$

$\dfrac{dy}{dx} = \dfrac{(\ln x)(6x) - (3x^2)\,1/x}{(\ln x)^2}$

$= \dfrac{6x\ln x - 3x}{(\ln x)^2}$

㉑ $y = (\underbrace{\ln|x+4|}_{g(x)})^4$

$\dfrac{dy}{dx} = 4(\ln|x+4|)^3 \;\dfrac{1}{x+4}$

(23) $y = \ln|\underbrace{\ln x}_{g(x)}|$

$$\frac{dy}{dx} = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x \ln x}$$

(25) $y = \underbrace{e^{x^2}}_{u} \underbrace{\ln x}_{v}$

$$\frac{dy}{dx} = e^{x^2}\left(\frac{1}{x}\right) + (\ln x)(2x\, e^{x^2})$$

(27)
$$y = \frac{e^x}{\ln x}$$
$$\frac{dy}{dx} = \frac{(\ln x)(e^x) - e^x(1/x)}{(\ln x)^2}$$

(29)

$$g(z) = (\underbrace{e^{2z} + \ln z}_{m(z)})^3$$

$$\left\{ m(z) = e^{2z} + \ln z \qquad m'(z) = 2e^{2z} + \frac{1}{x}\right.$$

$$g'(z) = 3(e^{2z} + \ln z)^2 \left(2e^{2z} + \frac{1}{x}\right)$$

(31)
$$y = \log(2x-3)$$
$$\Rightarrow y = \log_{10}(2x-3)$$
$$\left\{\begin{array}{l} y = \log_a g(x) \\ \frac{dy}{dx} = \frac{1}{\ln a}\frac{g'(x)}{g(x)} \end{array}\right.$$

$$\frac{dy}{dx} = \frac{1}{\ln(10)} \frac{2}{2x-3} = \frac{2}{(2x-3)\ln 10}$$

(33)
$$y = \log|3x|$$
$$\frac{dy}{dx} = \frac{1}{\ln 10} \cdot \frac{3}{3x} = \frac{1}{x \ln 10}$$

(35)
$$y = \log_7 \sqrt{2x-3}$$
$$y = \log_7 (2x-3)^{1/2}$$
$$y = 1/2 \log_7 (2x-3)$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{\ln 7}\frac{(2)}{(2x-3)}\right]$$

(37)
$$y = \log_2 (2x^2 - x)^{5/2}$$
$$y = \frac{5}{2}\log_2(2x^2 - x)$$

$$\frac{dy}{dx} = \frac{5}{2}\left[\frac{1}{\ln 2}\frac{(4x-1)}{(2x^2-x)}\right]$$

(39)
$$z = \underbrace{10^y}_{u}\,\underbrace{\log y}_{v}$$

$$\left\{\begin{array}{ll} \frac{dz}{dy} = uv' + vu' & \\ u = 10^y & v = \log y \\ u' = (\ln 10)(10^y) & v' = \frac{1}{\ln(10)}\frac{1}{y} \end{array}\right.$$

$$\frac{dz}{dy} = (10^y)\left(\frac{1}{y(\ln 10)}\right) + (\log y)(\ln 10)(10^y)$$

(49)
$$P(t) = \underbrace{(t+100)}_{u}\,\underbrace{\ln(t+2)}_{v}$$

$$P'(t) = (t+100)\left(\frac{1}{t+2}\right) + (\ln(t+2))(1)$$
$$P'(2) = (102)\left(\frac{1}{4}\right) + (\ln(4)) \approx 26.9$$
$$P'(8) = (108)\left(\frac{1}{10}\right) + \ln(12) \approx \underbrace{13.3}_{\text{answers may slightly vary}}$$