Math 151 Sect. 4.4 p. 237 [1-29, 37, 39] odd

① $y = e^{4x}$

$\begin{cases} y = e^{g(x)} \\ \frac{dy}{dx} = g'(x)\, e^{g(x)} \end{cases}$

$\begin{cases} g(x) = 4x & g'(x) = 4 \end{cases}$

$\frac{dy}{dx} = 4e^{4x}$

③ $y = -8e^{2x}$ $\begin{cases} g(x) = 2x \\ g'(x) = 2 \end{cases}$

$\frac{dy}{dx} = -8[2e^{2x}]$

$= -16e^{2x}$

⑤ $y = -16e^{(x+1)}$ $\begin{cases} g(x) = x+1 \\ g'(x) = 1 \end{cases}$

$\frac{dy}{dx} = -16[(1)e^{x+1}]$

$\frac{dy}{dx} = -16e^{x+1}$

⑦ $y = e^{(x^2)}$ $\begin{cases} g(x) = x^2 \\ g'(x) = 2x \end{cases}$

$\frac{dy}{dx} = 2xe^{x^2}$

⑨ $y = 3e^{(2x^2)}$ $\begin{cases} g(x) = 2x^2 \\ g'(x) = 4x \end{cases}$

$\frac{dy}{dx} = 3[4xe^{2x^2}]$

$= 12xe^{2x^2}$

⑪ $y = 4e^{2x^2-4}$ $\begin{cases} g(x) = 2x^2-4 \\ g'(x) = 4x \end{cases}$

$\frac{dy}{dx} = 4[4xe^{2x^2-4}]$

$= 16xe^{2x^2-4}$

⑬ $y = \underbrace{x}_{u}\underbrace{e^x}_{v}$

$\begin{cases} \frac{dy}{dx} = uv' + vu' \\ u = x \quad v = e^x \\ u' = 1 \quad v' = e^x \end{cases}$

$\frac{dy}{dx} = (x)(e^x) + (e^x)(1)$

$= xe^x + e^x$

⑮ $y = \underbrace{(x-3)^2}_{u}\underbrace{e^{2x}}_{v}$

$\begin{cases} \frac{dy}{dx} = uv' + vu' \\ u = (x-3)^2 \quad v = e^{2x} \\ u' = 2(x-3) \quad v' = 2e^{2x} \end{cases}$

$\frac{dy}{dx} = (x-3)^2(2e^{2x}) + (e^{2x})(2(x-3))$

⑰ $y = \frac{x^2 \leftarrow u}{e^x \leftarrow v}$

$\begin{cases} \frac{dy}{dx} = \frac{vu' - uv'}{v^2} \\ u = x^2 \quad v = e^x \\ u' = 2x \quad v' = e^x \end{cases}$

$\frac{dy}{dx} = \frac{e^x(2x) - (x^2)(e^x)}{(e^x)^2}$

$= \frac{2x - x^2}{e^x}$

⑲ $y = \frac{e^x + e^{-x} \leftarrow u}{x \leftarrow v}$

$\begin{cases} u' = e^x + (-1)e^{-x} & v' = 1 \end{cases}$

$\frac{dy}{dx} = \frac{x(e^x - e^{-x}) - (e^x + e^{-x})(1)}{x^2}$

㉑ $p = \frac{10{,}000 \leftarrow u}{9 + 4e^{-2t} \leftarrow v}$

$\begin{cases} u' = 0 & v' = -8e^{-2t} \end{cases}$

$\frac{dy}{dx} = \frac{(9+4e^{-2t})(0) - (10{,}000)(-8e^{-2t})}{(9+4e^{-2t})^2}$

$= \frac{80{,}000e^{-2t}}{(9+4e^{-2t})^2}$

㉓ $f(z) = (\underbrace{2z + e^{-z^2}}_{g(x)})^2$

$f'(x) = 2(2z + e^{-z^2})(2 + (-2ze^{-z^2}))$

$= 2(2z + e^{-z^2})(2 - 2ze^{-z^2})$

㉔ $y = 8^{5x}$ $\begin{cases} a = 8 \\ g(x) = 5x \end{cases}$ $g'(x) = 5$

$\begin{cases} y = a^{g(x)} \\ \frac{dy}{dx} = (\ln a)a^{g(x)}\, g'(x) \end{cases}$

$\frac{dy}{dx} = (\ln 8)8^{5x}(5)$

$= 5(\ln 8)8^{5x}$

(25) $y = 2^{-x}$ $\begin{cases} a = 2 \\ g(x) = -x \\ g'(x) = -1 \end{cases}$

$\frac{dy}{dx} = (\ln 2) 2^{-x} (-1)$

$= -(\ln 2) 2^{-x}$

(27) $y = -10^{3x^2-4} = (-1) 10^{3x^2-4}$ $\begin{cases} a = 10 \\ g(x) = 3x^2 - 4 \\ g'(x) = 6x \end{cases}$

$\frac{dy}{dx} = (-1)(\ln 10) 10^{3x^2-4} (6x)$

$\frac{dy}{dx} = -6x (\ln 10) 10^{3x^2-4}$

(29) $S = 5 \cdot 7^{\sqrt{t-2}} = 5 \cdot 7^{(t-2)^{1/2}}$ $\begin{cases} a = 7 \\ g(x) = (t-2)^{1/2} \\ g'(x) = \frac{1}{2}(t-2)^{-1/2}(1) \end{cases}$

$\frac{ds}{dt} = 5 \left[(\ln 7) 7^{(t-2)^{1/2}} \left(\frac{1}{2} (t-2)^{-1/2} \right) \right]$

$\frac{ds}{dt} = \frac{5}{2} (\ln 7) 7^{(t-2)^{1/2}} (t-2)^{-1/2}$