Solutions to Section 4.3 (MAT 151)

p. 229; [(23-41) odd, 45, 47, 49, 62]

(23)

$y = (\underbrace{2x^3+9x}_{g(x)})^{5}$ ; $y = [g(x)]^n$

$\frac{dy}{dx} = n[g(x)]^{n-1} \cdot g'(x)$

$g'(x) = 6x^2+9$

$\frac{dy}{dx} = 5[(2x^3+9x)]^4 \cdot (6x^2+9)$

(25)

$f(x) = -8(3x^4+2)^3$

$f'(x) = -8[3(3x^4+2)^2 \cdot (12x^3)]$

Simplify:

$f'(x) =$

(27)

$s(t) = 12(2t^4+5)^{3/2}$

$s'(t) = 12[\frac{3}{2}(2t^4+5)^{1/2}(8t^3)]$

Simplify:

(29)

$f(t) = 8\sqrt{4t^2+7} \xRightarrow{\text{Rewrite}} 8(4t^2+7)^{1/2}$

$f'(t) = 8[\frac{1}{2}(4t^2+7)^{-1/2}(8t)]$

Simplify:

(31)

$r(t) = \underbrace{4t}_{u}\underbrace{(2t^5+3)^2}_{v}$ : Product Rule

$u = 4t \quad v = (2t^5+3)^2$ : "Chain Rule"

$u' = 4 \quad v' = 2(2t^5+3)(10t^4)$

$r'(t) = uv' + vu'$

$= (4t)(2(2t^5+3)(10t^4)) + (2t^5+3)^2(4)$

(33)

$y = \underbrace{(x^3+2)}_{u}\underbrace{(x^2-1)^2}_{v}$ : Product Rule

$u = x^3+2 \quad v = (x^2-1)^2$ : "Chain Rule"

$u' = 3x^2 \quad v' = 2(x^2-1)(2x)$

$\frac{dy}{dx} = uv' + vu'$

$= (x^3+2)(2(x^2-1)(2x)) + (x^2-1)^2(3x^2)$

(35)

$p(z) = \underbrace{z}_{u}\underbrace{(6z+1)^{4/3}}_{v}$ : Product Rule

$u = z \quad v = (6z+1)^{4/3}$

$u' = 1 \quad v' = \frac{4}{3}(6z+1)^{1/3}(6)$

$p'(z) = uv' + vu'$

$= (z)(\frac{4}{3}(6z+1)^{1/3}(6)) + (6z+1)^{4/3}(1)$

$= (8z)(6z+1)^{1/3} + (6z+1)^{4/3}$

(37)

$y = \frac{1 \leftarrow u}{(3x^2-4)^5 \leftarrow v}$ [Alt. $y = (3x^2-4)^{-5}$]

$u = 1 \quad v = (3x^2-5)^5$

$u' = 0 \quad v' = 5(3x^2-5)^4(6x)$

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

$= \frac{(3x^2-5)^5(0) - (1)(5(3x^2-5)^4(6x))}{((3x^2-4)^5)^2}$

$= \frac{-5(3x^2-5)^4(6x)}{(3x^2-4)^{10}} = \frac{-30x}{(3x^2-4)^6}$

(39)
$$p(t) = \frac{(2t+3)^3 \leftarrow u}{4t^2-1 \leftarrow v} \text{ : Quotient Rule}$$

$u = (2t+3)^3 \qquad v = 4t^2-1$

$u' = 3(2t+3)^2(2) \qquad v' = 8t$

$u' = 6(2t+3)^2$

$$p'(t) = \frac{vu' - uv'}{v^2}$$
$$= \frac{(4t^2-1)(6(2t+3)^2) - (2t+3)^3(8t)}{(4t^2-1)^2}$$

(41)
$$y = \frac{x^2+4x \leftarrow u}{(3x^3+2)^4 \leftarrow v} \text{ : Quotient Rule}$$

$u = x^2+4x \qquad v = (3x^3+2)^4$

$u' = 2x+4 \qquad v' = 4(3x^3+2)^3(9x^2)$

$$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$$
$$= \frac{(3x^3+2)^4(2x+4) - (x^2+4x)[4(3x^3+2)^3(9x^2)]}{((3x^3+2)^4)^2}$$

(45)
$f(x) = \sqrt{x^2+16}$ ; $x=3$

$f(x) = (x^2+16)^{1/2}$ ; $x=3$

$$f'(x) = \tfrac{1}{2}(x^2+16)^{-1/2}(2x)$$
$$= \frac{x}{(x^2+16)^{1/2}}$$

$y = f(3) = (3^2+16)^{1/2} = 5$

$$m_{tan} = f'(3) = \frac{3}{(3^2+16)^{1/2}} = \frac{3}{5}$$

$y = m_{tan}x + b \Rightarrow y = \frac{3}{5}x + \frac{16}{5}$

$5 = \frac{3}{5}(3) + b$

$5 = \frac{9}{5} + b$

$\frac{16}{5} = b$

(47)
$f(x) = \underbrace{x}_{u}\underbrace{(x^2-4x+5)^4}_{v}$ ; $x=2$

$y = f(2) = 2(2^2-4(2)+5)^4 = 2$

$f'(x) = x[4(x^2-4x+5)^3(2x-4)] + (x^2-4x+5)^4(1)$

$m_{tan} = f'(2) = 2[4(2^2-4(2)+5)^3(2(2)-4)] + (2^2-4(2)+5)^4$

slope of tangent line : $m_{tan} = f'(2) = 1$

$y = m_{tan}x + b \Rightarrow y = x$

$2 = (1)(2) + b$

$0 = b$

(49)
$f(x) = (x^3-6x^2+9x+1)^{1/2}$

$$f'(x) = \tfrac{1}{2}(x^3-6x^2+9x+1)^{-1/2}(3x^2-12x+9)$$
$$= \frac{3x^2-12x+9}{2(x^3-6x^2+9x+1)^{1/2}}$$

$f'(x) = 0$, find $x$-values :

$$\frac{3(x^2-4x+3)}{2(x^3-6x^2+9x+1)^{1/2}} = 0$$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x = 3 \qquad x = 1$

(62)
$N(t) = 2t(5t+9)^{1/2} + 12$

$N'(t) = 2t(\tfrac{1}{2}(5t+9)^{-1/2}(5)) + (5t+9)^{1/2}(2) + 0$

$= 5t(5t+9)^{-1/2} + 2(5t+9)^{1/2}$

$N'(0) =$

$N'(7/5) =$

$N'(8) =$