Math 151 Sect. 4.2 p. 219; [1-25 (odd), 26, 27, 29, 31, 41]

Product Rule:

$\frac{dy}{dx} = uv' + vu'$

Quotient Rule:

$\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

① $\{u' = 6x \quad v' = 2\}$

$y = \underbrace{(3x^2+2)}_{u}\underbrace{(2x-1)}_{v}$: P.R.

$\frac{dy}{dx} = (3x^2+2)(2) + (2x-1)(6x)$

Simplify: $\frac{dy}{dx} = 6x^2 + 4 + 12x^2 - 6x$

$\boxed{\frac{dy}{dx} = 18x^2 - 6x + 4}$

③ $\{u' = 2 \quad v' = 2\}$

$y = (2x-5)^2 = \underbrace{(2x-5)}_{u}\underbrace{(2x-5)}_{v}$

$\boxed{\frac{dy}{dx}} = (2x-5)(2) + (2)(2x-5)$

$= 4(2x-5) = \boxed{8x - 20}$

OR $4x - 10 + 4x - 10 = 8x - 20$

⑤ $\{u' = 2t \quad v' = 2t\}$

$k(t) = (t^2-1)^2 = \underbrace{(t^2-1)}_{u}\underbrace{(t^2-1)}_{v}$

$\boxed{k'(t) =} (t^2-1)(2t) + (t^2-1)(2t)$

$= 4t(t^2-1) = 4t^3 - 4t$ OR

$= 2t^3 - 2t + 2t^3 - 2t = \boxed{4t^3 - 4t}$

⑦ $\{u' = 1 \quad v' = \frac{1}{2}x^{-1/2}\}$

$y = (x+1)(\sqrt{x}+2) = \underbrace{(x+1)}_{u}\underbrace{(x^{1/2}+2)}_{v}$

$\frac{dy}{dx} = (x+1)(\frac{1}{2}x^{-1/2}) + (x^{1/2}+2)(1)$

$= \frac{1}{2}x^{1/2} + \frac{1}{2}x^{-1/2} + x^{1/2} + 2$

$\boxed{\frac{dy}{dx} = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2} + 2}$

⑨

$p(y) = \underbrace{(y^{-1} + y^{-2})}_{u}\underbrace{(2y^{-3} - 5y^{-4})}_{v}$

$$\begin{cases} u = y^{-1} + y^{-2} & v = 2y^{-3} - 5y^{-4} \\ u' = -y^{-2} + -2y^{-3} & v' = -6y^{-4} - (-20y^{-5}) \\ \quad = -y^{-2} - 2y^{-3} & \quad = -6y^{-4} + 20y^{-5} \end{cases}$$

$p'(y) = (y^{-1} + y^{-2})(-6y^{-4} + 20y^{-5}) + (2y^{-3} - 5y^{-4})(-y^{-2} - 2y^{-3})$

Quotient Rule: $\frac{dy}{dx} = \frac{vu' - uv'}{v^2}$

(11)
$$f(x) = \frac{7x+1 \leftarrow u;\ u' = 7}{3x+8 \leftarrow v;\ v' = 3}$$

$$\begin{cases} u = 7x+1 & v = 3x+8 \\ u' = 7 & v' = 3 \end{cases}$$

$$f'(x) = \frac{(3x+8)(7) - (7x+1)(3)}{(3x+8)^2}$$

$$= \frac{21x + 56 - 21x - 3}{(3x+8)^2}$$

$$\boxed{f'(x) = \frac{53}{(3x+8)^2}}$$

(13)
$$y = \frac{5-3t \leftarrow u;\ u' = -3}{4+t \leftarrow v;\ v' = 1}$$

$$\begin{cases} u = 5-3t & v = 4+t \\ u' = 0-3 = -3 & v' = 1 \end{cases}$$

$$\frac{dy}{dx} = \frac{(4+t)(-3) - (5-3t)(1)}{(4+t)^2}$$

$$= \frac{-12 - 3t - 5 + 3t}{(4+t)^2}$$

$$\boxed{\frac{dy}{dx} = \frac{-17}{(4+t)^2}}$$

(15)
$$y = \frac{x^2+x \leftarrow u;\ u' = 2x+1}{x-1 \leftarrow v;\ v' = 1}$$

$$\begin{cases} u = x^2+x & v = x-1 \\ u' = 2x+1 & v' = 1 \end{cases}$$

$$\frac{dy}{dx} = \frac{(x-1)(2x+1) - (x^2+x)(1)}{(x-1)^2}$$

(17)
$$f(t) = \frac{4t+11 \leftarrow u;\ u' = 4}{t^2-3 \leftarrow v;\ v' = 2t}$$

$$\begin{cases} u = 4t+11 & v = t^2-3 \\ u' = 4 & v' = 2t \end{cases}$$

$$f'(t) = \frac{(t^2-3)(4) - (4t+11)(2t)}{(t^2-3)^2}$$

$$= \frac{(4t^2-12) - (8t^2+22t)}{(t^2-3)^2}$$

$$= \frac{4t^2 - 12 - 8t^2 - 22t}{(t^2-3)^2}$$

$$= \frac{-4t^2 - 22t - 12}{(t^2-3)^2}$$

(19)
$$g(x) = \frac{x^2-4x+2 \leftarrow u;\ u' = 2x-4}{x+3 \leftarrow v;\ v' = 1}$$

$$\begin{cases} u = x^2-4x+2 & v = x+3 \\ u' = 2x-4 & v' = 1 \end{cases}$$

$$g'(x) = \frac{(x+3)(2x-4) - (2x-4)(1)}{(x+3)^2}$$

(21)
$$p(t) = \frac{\sqrt{t}}{t-1} = \frac{t^{1/2} \leftarrow u;\ u' = \frac{1}{2}t^{-1/2}}{t-1 \leftarrow v;\ v' = 1}$$

$$\begin{cases} u = t^{1/2} & v = t-1 \\ u' = \frac{1}{2}t^{-1/2} & v' = 1 \end{cases}$$

$$p'(t) = \frac{(t-1)(\frac{1}{2}t^{-1/2}) - (t^{1/2})(1)}{(t-1)^2}$$

$$= \frac{\frac{1}{2}t^{1/2} - \frac{1}{2}t^{-1/2} - t^{1/2}}{(t-1)^2} = \frac{-\frac{1}{2}t^{1/2} - \frac{1}{2}t^{-1/2}}{(t-1)^2}$$

(23)

$$y = \frac{5x+6}{\sqrt{x}} = \frac{5x+6 \leftarrow u}{x^{1/2} \leftarrow v}$$

$$\begin{cases} u = 5x+6 & v = x^{1/2} \\ u' = 5 & v' = \frac{1}{2}x^{-1/2} \end{cases}$$

$$\frac{dy}{dx} = \frac{x^{1/2}(5) - (5x+6)(\frac{1}{2}x^{-1/2})}{(x^{1/2})^2}$$

$$= \frac{\underline{5x^{1/2} - \frac{5}{2}x^{1/2} - 3x^{-1/2}}}{x}$$

$$= \frac{\frac{5}{2}x^{1/2} - 3x^{-1/2}}{x}$$

(25)

$$g(y) = \frac{y^{1.4} + 1 \leftarrow u}{y^{2.5} + 2 \leftarrow v}$$

$$\begin{cases} u = y^{1.4} + 1 & v = y^{2.5} + 2 \\ u' = 1.4y^{0.4} & v' = 2.5y^{1.5} \end{cases}$$

$$g'(x) = \frac{(y^{2.5}+2)(1.4y^{0.4}) - (y^{1.4}+1)(2.5y^{1.5})}{(y^{2.5}+2)^2}$$

$$g'(x) = \frac{1.4y^{2.9} + 2.8y^{0.4} - 2.5y^{2.9} - 2.5y^{1.5}}{(y^{2.5}+2)^2}$$

$$= \frac{-1.1y^{2.9} - 2.5y^{1.5} + 2.8y^{0.4}}{(y^{2.5}+2)^2}$$

(27)

$$h(x) = f(x)g(x)$$

$$h'(x) = f(x)g'(x) + g(x)f'(x)$$

$$h'(3) = f(3)g'(3) + g(3)f'(3)$$

$$= (7)(5) + (4)(6)$$

$$= 35 + 24 = 59$$

(29)

$$D_x\left(\frac{u}{v}\right) \neq \frac{uv' - vu'}{(v^2)}$$

$$D_x\left(\frac{u}{v}\right) = \frac{vu' - uv'}{(v^2)}$$

(31)

$$f(x) = \frac{x}{x-2} \text{ at } (3,3) \begin{smallmatrix} \nearrow x = 3 \\ \searrow y = 3 \end{smallmatrix}$$

$f'(3) = ?$ (slope of the tangent line)

$$f'(x) = \frac{(x-2)(1) - (x)(1)}{(x-2)^2}$$

$$= \frac{x-2-x}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

$$f'(3) = \frac{-2}{(3-2)^2} = \frac{-2}{(1)^2} = -2$$

Using $y = m_{tan}x + b \Rightarrow \boxed{y = -2x + 9}$

$$3 = (-2)(3) + b$$

$$3 = -6 + b$$

$$9 = b$$

(41)

$$N(t) = (t-10)^2(2t) + 50$$

$$= \underbrace{(t^2 - 20t + 100)(2t)}_{u} + \underbrace{50}_{v}$$

$$\begin{cases} u = \underbrace{(t^2 - 20t + 100)}_{a}\underbrace{(2t)}_{b} & v = 50 \\ & v' = 0 \\ u' = (t^2 - 20t + 100)(2) + (2t)(2t - 20) & \end{cases}$$

$$N'(t) = 2t^2 - 40t + 200 + 4t^2 - 40t + 0$$

a) $N'(t) = 6t^2 - 80t + 200$

b) $N'(8) = 6(8)^2 - 80(8) + 200$

$= -56$ million/hr.

c) $N'(11) = 6(11)^2 - 80(11) + 200$

$= 46$ million/hr.