Section 4.4 Derivatives of Exponential and Logarithmic Functions

1. $f'(x) = 4e^{4x}$

3. $f'(x) = -3e^{-3x}$

5. $f'(x) = (4x)e^{2x^2+1}$

7. $f'(x) = \frac{1}{2}x^{-1/2}e^{\sqrt{x}}$

9. $f'(x) = x^2e^x + 2xe^x$

11. $f'(x) = x^5e^x + 5x^4e^x$

13. $f'(x) = \dfrac{e^x(1) - xe^x}{(e^x)^2}$

$= \dfrac{1-x}{e^x}$

15. $f'(x) = x^2(2xe^{x^2}) + 2xe^{x^2}$

$= 2xe^{x^2}(x^2+1)$

17. $f'(x) = \frac{1}{2}(e^x+1)^{-1/2}e^x$

19. $f'(x) = x^{1/2}\left(\frac{1}{2}x^{-1/2}e^{x^{1/2}}\right) + \left(\frac{1}{2}x^{-1/2}\right)e^{x^{1/2}}$

21. $f'(x) = \dfrac{(1+e^x)(1) - x(e^x)}{(1+e^x)^2}$

23. $f'(x) = \dfrac{(e^{2x}+e^{-2x})(2e^{2x}+2e^{-2x}) - (e^{2x}-e^{-2x})(2e^{2x}-2e^{-2x})}{(e^{2x}+e^{-2x})^2}$

25. $f'(x) = \frac{1}{2}(e^{3x}+2)^{-1/2}(3e^{3x})$

27. $f'(x) = (\ln 5)(5^x)$

29. $f'(x) = (\ln 3)(3^{x^{1/2}})(\frac{1}{2}x^{-1/2})$

31. $f'(x) = x(\ln 3)3^x + 3^x(1)$

33. $f'(x) = \dfrac{3x^2+2x}{x^3+x^2+1}$

35. $f'(x) = x^2\left(\frac{2}{x}\right) + 2x\ln x^2$

37. $f'(x) = \frac{1}{2}(\ln|x^2+x+1|)^{-1/2} \cdot \dfrac{(2x+2)}{x^2+x+1}$

39. $f'(x) = \dfrac{1}{x} - \dfrac{1}{x+1}$

41. $f'(x) = e^{-x^2}\left(\frac{2}{x}\right) + -2xe^{-x^2}\ln x^2$

43. $y' = x^2(\frac{1}{x}) + 2x\ln x$

45. $y' = \dfrac{(x^2+1)(1/x) - (\ln x)(2x)}{(x^2+1)^2}$

47. $y' = 11(\ln x)^{10}(\frac{1}{x})$

49. $y' = \dfrac{3}{x}$

51. $y' = x(-3e^{-3x}) + e^{-3x}(1)$

$= e^{-3x}(-3x+1)$ $\quad e^{-3x}(-3x+1) = 0$

$e^{-3x} > 0$; $-3x+1=0 \rightarrow x = 1/3$

53. $y' = -2xe^{-x^2}$

$-2xe^{-x^2} = 0$ $\quad e^{-x^2} > 0$; $-2x = 0 \rightarrow x = 0$

55. $y' = x^2e^x + 2xe^x$

$xe^x(x+2) = 0$

$x = 0$; $e^x > 0$; $x+2=0 \rightarrow x = -2$

57. $y' = x^2(-e^{-x}) + 2xe^{-x}$

$= xe^{-x}(-x+2) = 0$

$x = 0$; $e^{-x} > 0$; $-x+2=0 \rightarrow x = 2$

59. $y' = x\left(\frac{2}{x}\right) + \ln x^2$

$= 2 + 2\ln x$

$2 + 2\ln x = 0$ $\rightarrow$ $2\ln x = -2$

$\ln x = -1$

$x = e^{-1}$

61. $y' = \dfrac{x^2(1/x) - 2x(\ln x)}{x^4}$

$= \dfrac{1 - 2\ln x}{x^3}$

$1 - 2\ln x = 0$

$2\ln x = -1$

$\ln x = -1/2$ $\quad x = e^{-1/2}$

63. $R'(x) = x(-2e^{-2x}) + 1e^{-2x} = e^{-2x}(-2x+1)$

$-2x+1=0 \rightarrow x = 1/2$

65. $C'(x) = \frac{2x}{x^2+5} > 0$

$2x = 0$ $\quad$ x^2+5 always +

$x = 0$

$C'(x)$ $\quad \ominus \quad | \quad \oplus$

$-1 \quad 0 \quad 1$

$C'(-1) = -\frac{2}{6}$ $\quad$ $C'(1) = +\frac{2}{6}$

$C'(x)$ is positive over the interval $(0, \infty)$.

71. $f'(x) = \frac{1}{\ln 2} \frac{3e^{3x} + 2x}{e^{3x} + x^2}$