4.3 The Chain Rule

1. $f'(x) = 7(2x+1)^6(2)$
$= 14(2x+1)^6$

3. $f'(x) = 4(x^2+1)^3(2x)$
$= 8x(x^2+1)^3$

5. $f'(x) = \frac{3}{2}(2e^x-3)^{1/2}(2e^x)$
$= 3e^x(2e^x-3)^{1/2}$

7. $f'(x) = \frac{1}{2}(e^x)^{-1/2}(e^x)$
$= \frac{1}{2}e^{\frac{1}{2}x} = \frac{1}{2}\sqrt{e^x}$

9. $f'(x) = \frac{1}{2}(2x+1)^{-1/2}(2)$
$= (2x+1)^{-1/2}$

11. $f'(x) = 2(x^2+1)^{-2/3}(2x)$
$= 4x(x^2+1)^{-2/3}$

13. $f'(x) = -1(e^x+1)^{-2}e^x$
$= -e^x(e^x+1)^{-2}$

15. $f'(x) = -2[-1(x^3+1)^{-2}(3x^2)]$
$= 6x^2(x^3+1)^{-2}$

17. $f'(x) = \dfrac{(x^2+1)^{1/2}(1/x) - (\ln x)(\frac{1}{2}(x^2+1)^{-1/2}(2x))}{x^2+1}$
$= \dfrac{(x^2+1)^{1/2}(1/x) - (\ln x)(x(x^2+1)^{-1/2})}{x^2+1}$

19. $f'(x) = (2x+1)(5(3x-2)^4(3)) + (3x-2)^5(2)$
$= (2x+1)(15)(3x-2)^4 + 2(3x-2)^5$
$= (3x-2)^4[(2x+1)(15) + 2(3x-2)]$
$= (3x-2)^4(36x+11)$

21. $f'(x) = 4(\ln x)^3(1/x)$

23. $\frac{dy}{dx} = e^x[8(x^2+1)^7(2x)] + (x^2+1)^8e^x$
$= e^x(x^2+1)^7[x^2+16x+1]$

25. $y' = (1-x)^4[5(2x-1)^4(2)] + (2x-1)^5[4(1-x)^3(-1)]$
$= (1-x)^3(2x-1)^4[(1-x)(10) + (2x-1)(-4)]$
$10-10x \quad -8x+4$
$= -2(1-x)^3(2x-1)^4(9x-7)$

27. $f'(x) = (x^{1/2})(3(4x+3)^2(4)) + (4x+3)^3(\frac{1}{2}x^{-1/2})$
$= (4x+3)^2[12x^{1/2} + \frac{1}{2}x^{-1/2}(4x+3)]$
$= (4x+3)^2\left(\frac{28x+3}{2x^{1/2}}\right)$

29. $f'(x) = e^x(\ln x)^2$
$= e^x[2(\ln x)(\frac{1}{x})] + (\ln x)^2e^x$
$= e^x\ln x[\frac{2}{x} + \ln x]$

31. $\frac{dy}{dx} = 3(-4(x+1)^{-5}(1))$
$= -12(x+1)^{-5}$

33. $\frac{dy}{dx} = \dfrac{(2x+3)^3(e^x) - (e^x+1)[3(2x+3)^2(2)]}{(2x+3)^6}$
$= \dfrac{(2x+3)(e^x) - (e^x+1)(6)}{(2x+3)^4}$
$= \dfrac{2xe^x - 3e^x - 6}{(2x+3)^4}$

35. $f'(x) = \frac{1}{2}(\sqrt{x}+1)^{-1/2}\frac{1}{2}x^{-1/2}$
$= \dfrac{1}{4\sqrt{\sqrt{x}+1}\sqrt{x}} = \dfrac{1}{4\sqrt{x(\sqrt{x}+1)}}$

41. $\frac{dV}{dn} = \frac{4\pi r^3}{3}\left(3\left(1+\frac{c}{r}n^{1/3}\right)^2\left(\frac{1}{3}\frac{c}{r}n^{-2/3}\right)\right)$
$= \frac{4\pi r^3}{3}\left(1+\frac{c}{r}n^{1/3}\right)^2\left(\frac{c}{r}n^{-2/3}\right)$
$= \dfrac{4\pi cr^2}{3n^{2/3}}\left(1+\frac{c}{r}n^{1/3}\right)^2$