Math 151 *Calculus with Applications and Technology* Name Key

Final Exam DATE: **TIME:** **PLACE:**

~Final Exam **Mini**-Review~ Remember to study your old tests and recommended homework.

INSTRUCTIONS: You must show your work to validate your answers.

1. Given $R(x) = 4x^2 - 7x$ and $C(x) = 3x^2 - x + 7$.
   What is/are the break-even quantity/quantities?
   What is the profit function?

2. Given $f(x) = x^2 - 8x + 9$ and $f'(x) = 2x - 8$.
   Critical number(s)?
   Largest interval(s) increasing?
   Largest interval(s) decreasing?

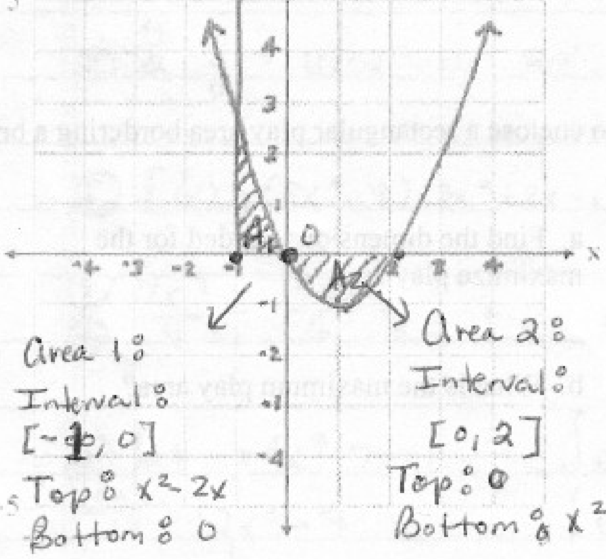
3. Find the derivatives of the following functions.
   a. $f(x) = 8x^5 - 19x^2 + x - 23$ b. $f(x) = \sqrt{2x^3 - 6x + 11}$
   b. $y = (e^{3x^2-9x+2})(\ln(4x^5 - 7x + 11))$ d. $f(x) = \dfrac{6x^4 - x^3 + 17}{x - 13}$
   e. $y = (5x^7 - 17x^2 - 8x)^{11}$ e. $f(x) = (2x^4 - 9)(x^3 + x^2 - x)$

4. Find the antiderivative of the following functions.
   a. $\int (7x^8 - 19x^5 + x + 8)dx$
   b. $\int (e^{4x^3-7x+1})(12x^2 - 7)dx$

5. What is $f'(x)$ according to the limit definition of the derivative?

6. Evaluate the following antiderivative.
   $\int_{-4}^{6} (x^3 - x + 8)dx$

7. Find the area between the curves. $y = x^2 - 2x$ and $y = 0$; interval [-1,2].



8. Find the limits of the following functions.

a. $\lim_{x\to 9}(11x+10)$

c. $\lim_{x\to\infty}\frac{\cancel{x}-\cancel{3x^4}+81x^5}{9x^5-\cancel{4x^2}-\cancel{x+5}}$

$\frac{0}{0}$ b. $\lim_{x\to 5}\frac{x^2-2x-15}{x-5} \rightarrow \frac{(x-5)(x+3)}{(x-5)} = 5+3$ $\boxed{8}$

d. $\lim_{x\to\infty}\frac{27x^3-\cancel{5x^2}-\cancel{9}}{9x^{11}-\cancel{8x+1}}$

9. Find the equation of the tangent line given the function $f(x)=x^3+3x^2-1$ when $x=1$.

10. Find the ***absolute maximum*** and/or ***absolute minimum*** value for the following function.

$f(x)=x^3-3x^2-24x+5;\ [-3,6]$

~~Relative~~ max. of ____ at $x=$ _____. ~~Relative~~ min. of ____ at $x=$ ______.

Absolute Absolute

11. Use the graph to draw a sketch representing the following for the function, $f(x)$. Fill-in the blank for part d.

If the limit DNE, explain why:

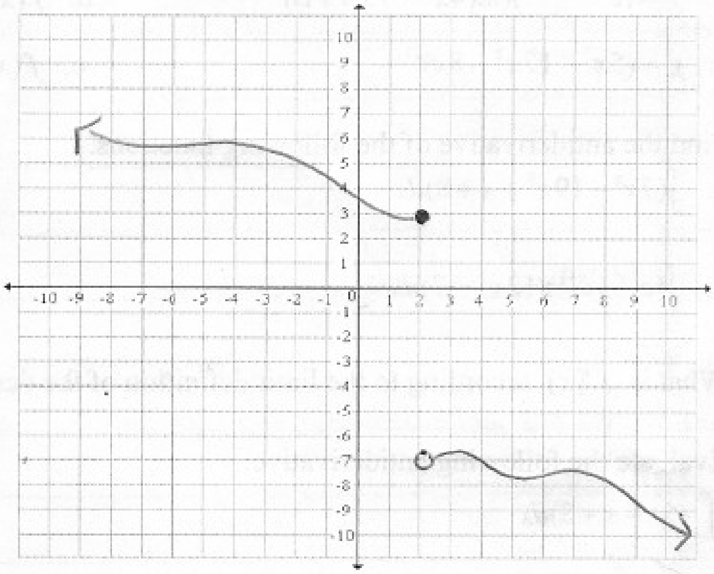
a. $\lim_{x\to 2^-} f(x)=3$

b. $\lim_{x\to 2^+} f(x)=-7$

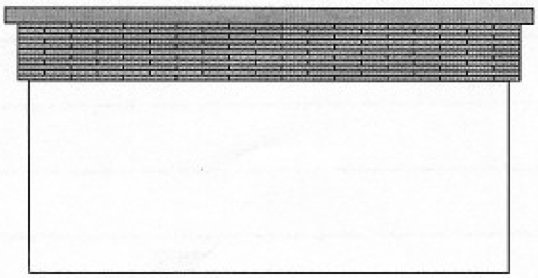
c. $f(2)=3$

d. $\lim_{x\to 2} f(x)=$ DNE

$\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$



12. ***Area*** A family has 1200 m of fencing. They want to enclose a rectangular play area bordering a brick wall. No fencing is needed along the brick wall.



x

y

a. Find the dimensions needed for the maximize play area.

b. What is the maximum play area?

1

Break-Even:

$R(x) = C(x)$
or
$R(x) - C(x) = 0$

$4x^2 - 7x - (3x^2 - x + 7) = 0$

$4x^2 - 7x - 3x^2 + x - 7 = 0$

$x^2 - 6x - 7 = 0$

$(x-7)(x+1) = 0$

$\boxed{x=7}$ $x=-1$

Profit function:

$P(x) = R(x) - C(x)$

$P(x) = x^2 - 6x - 7$

2 $f(x)$: Domain $\mathbb{R}$

$f'(x) = 2x - 8 = 0$

$2x - 8 = 0$

$x = 4$

C#: 4

Inc. ↗: $(4, \infty)$

Dec. ↘: $(-\infty, 4)$

$-$ ↘ | 4 | ↗ $+$

$f'(0) = 2(0) - 8 = -8$ $(-)$

$f'(5) = 2(5) - 8 = 2$ $(+)$

3

a) $f'(x) = 40x^4 - 38x + 1$

b) $f'(x) = \frac{1}{2}(2x^3 - 6x + 11)^{-1/2}(6x^2 - 6)$

c) $\frac{dy}{dx} = \underset{u}{e^{3x^2-9x+2}}\underset{v'}{\left(\frac{20x^4 - 7}{4x^5 - 7x + 11}\right)} \underset{+}{+} \underset{v}{\ln(4x^5 - 7x + 11)}\underset{u'}{(6x-9)e^{3x^2-9x+2}}$

d) $f'(x) = \frac{(x-13)(24x^3 - 3x^2) - (6x^4 - x^3 + 17)(1)}{(x-13)^2}$

e) $\frac{dy}{dx} = y' = 11(5x^7 - 17x^2 - 8x)^{10}(35x^6 - 34x - 8)$

f) $f'(x) = (2x^4 - 9)(3x^2 + 2x - 1) + (x^3 + x^2 - x)(8x^3)$

4

a) $\frac{7x^9}{9} - \frac{19x^6}{6} + \frac{x^2}{2} + 8x + C$

b) Let $u = 4x^3 - 7x + 1$

$\frac{du}{dx} = 12x^2 - 7$

$dx = \frac{du}{12x^2 - 7}$

$\int e^u (12x^2 - 7)\frac{du}{12x^2 - 7}$

$\int e^u \, du$

$e^u + C$

Solution:

$e^{4x^3 - 7x + 1} + C$

(5)

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

(6)

$\frac{x^4}{4} - \frac{x^2}{2} + 8x \Big|_{-4}^{5}$ $F(5) - F(-4) =$

$F(5) = \frac{(5)^4}{4} - \frac{(5)^2}{2} + 8(5) = 183.75$ $\boxed{159.75}$

$F(-4) = \frac{(-4)^4}{4} - \frac{(-4)^2}{2} + 8(-4) = 24$

(7)

$A_1: \int_{-1}^{0} (x^2-2x)-(0)\,dx$

$\int_{-1}^{0} x^2-2x\,dx$

$= \frac{x^3}{3} - x^2 \Big|_{-1}^{0}$

$F(0) = \frac{(0)^3}{3} - (0)^2 = 0$

$F(-1) = \frac{(-1)^3}{3} - (-1)^2 = -\frac{4}{3}$

$F(0) - F(-1) = 0 - (-4/3) = \frac{4}{3}$

$A_2 = \int_{0}^{2} (0) - (x^2-2x)\,dx$

$= \int_0^2 -x^2 + 2x\,dx$

$= \frac{-x^3}{3} + x^2 \Big|_0^2$

$F(2) = \frac{-(2)^3}{3} + (2)^2 = 4/3$

$F(0) = \frac{-(0)^3}{3} + (0)^2 = 0$

$F(2) - F(0) = 4/3 - 0 = 4/3$

(*) $A_{Total} = A_1 + A_2 = 4/3 + 4/3 = \boxed{8/3}$

(8)

(a) 109 (b) 8 (c) 9 (d) 0 Remember: $0 \neq$ DNE

(9)

$f(x) = x^3 + 3x^2 - 1 \rightarrow$ When $x=1$, $y = f(1) = (1)^3 + 3(1)^2 - 1 = 3$ POINT: $(1,3)$

$f'(x) = 3x^2 + 6x \longrightarrow$ When $x=1$, $m_T\big|_{x=1} = f'(1) = 3(1)^2 + 6(1) = 9$ Slope: 9

$y = m_T x + b$ OR $y - y_1 = m_T(x - x_1)$

$3 = 9(1) + b$ $y - 3 = 9(x-1)$

$-6 = b$ $y - 3 = 9x - 9$

$y = 9x - 6$ $y = 9x - 6$

(10)

$f'(x) = 3x^2 - 6x - 24$

$= 3(x^2 - 2x - 8)$

C#: $3(x-4)(x+2) = 0$

$x = 4$ $x = -2$

Both critical numbers fall within the given interval $[-3,6]$.

endpoints and critical #'s ONLY.

	x	$f(x) = x^3 - 3x^2 - 24x + 5$
[EP]	-3	$f(-3) = (-3)^3 - 3(-3)^2 - 24(-3) + 5 = 23$
[C#]	-2	$f(-2) = 33$
[C#]	4	$f(4) = -75$
[EP]	6	$f(6) = -31$

Absolute Maximum of 33 when $x = -2$,

Absolute Minimum of -75 when $x = 4$.

(11) Graphs may vary.

(12) $2x + y = 1200$ $\quad x \geq 0$ $\quad [0, 600]$

$A = xy$ $\quad y = 1200 - 2x \geq 0$

$-2x \geq -1200$

$x \leq 600$

$y = 1200 - 2x$

$A(x) = x(1200 - 2x)$

$A(x) = 1200x - 2x^2$ (Parabola) $\quad A''(x) = -4$ (always $\cap$)

$A'(x) = 1200 - 4x$ $\quad$ So $A(300) = 1200(300) - 2(300)^2 =$

$1200 - 4x = 0$ $\quad y = 1200 - 2(300)$

$4x = 1200$ $\quad = 600$

$x = 300$ $\quad$ or $A_{max} = (300)(600) = 180{,}000 m^2$

Dimensions: x by y

300 m by 600 m

OR

x	$A(x) = x(1200 - 2x) = 1200x - 2x^2$
0	$A(0) = 0$
x $\boxed{300}$	$A(300) = \boxed{180{,}000}$ ← A_{max}
600	$A(600) = 0$

$y = 1200 - 2(300) = \boxed{600}\ y$