Fall 2005 Section 6.2 1-33 (ODD), 35,

①
$\int 6(3x+1)^{10}\,dx$

$u = 3x+1$

$\frac{du}{dx} = 3$

$dx = \frac{du}{3}$

$\int 6(u^{10})\frac{du}{3}$

$2\int u^{10}\,du$

$2\left(\frac{u^{11}}{11}\right)+C$

$= \frac{2(3x+1)^{11}}{11} + C$

③
$\int x(3-x^2)^7\,dx = \int (3-x^2)\,x\,dx$

$u = 3-x^2$

$\frac{du}{-2} = \frac{-2x\,dx}{-2}$

$\frac{du}{-2} = x\,dx$

$\int (u)\frac{du}{-2}$

$-\frac{1}{2}\int u\,du$

$-\frac{1}{2}\left(\frac{u^2}{2}\right)+C$

$-\frac{1}{4}(3-x^2)+C$

⑤
$\int 8(x+1)(2x^2+4x-1)^{3/2}\,dx$

$u = 2x^2+4x-1$

$\frac{du}{dx} = 4x+4 = 4(x+1)$

$\frac{du}{4(x+1)} = dx$

$\int \overset{2}{\cancel{8}}\cancel{(x+1)}(u)^{3/2}\frac{du}{\cancel{4(x+1)}}$

$2\int u^{3/2}\,du$

$2\left(\frac{u^{5/2}}{5/2}\right)+C$

$\frac{4u^{5/2}}{5}+C$

$= \frac{4(2x^2+4x-1)^{5/2}}{5} + C$

⑦
$\int (x^3+x^2+x+1)(3x^4+4x^3+6x^2+12x+1)^3\,dx$

$\frac{du}{dx} = 12x^3+12x^2+12x+12$

$\frac{du}{12(x^3+x^2+x+1)} = dx$

$\int (x^3+x^2+x+1)\,u^3\,\frac{du}{12(x^3+x^2+x+1)}$

$\frac{1}{12}\int u^3\,du$

$\frac{1}{12}\left(\frac{u^4}{4}\right)+C$

$= \frac{1}{48}(3x^4+4x^3+6x^2+12x+1)^4 + C$

⑨
$\int 2(x+1)^{1/2}\,dx$

$u = x+1 \qquad \frac{du}{dx} = 1$

$du = dx$

$2\int u^{1/2}\,dx$

$2\left(\frac{u^{3/2}}{3/2}\right)+C$

$\frac{4}{3}u^{3/2}+C$

$= \frac{4}{3}(x+1)^{3/2}+C$

⑪
$\int x(x^2+1)^{1/2}\,dx$

$u = x^2+1 \qquad \frac{du}{2} = \frac{2x\,dx}{2}$

$\frac{du}{2} = x\,dx$

$\frac{1}{2}\int (u^{1/2})\,du$

$\frac{1}{2}\left(\frac{u^{3/2}}{3/2}\right)+C$

$3u^{3/2}+C$

$= 3(x^2+1)^{3/2}+C$

⑬
$\int x\underbrace{(x^2+1)}_{u}^{-1/3}\,dx$

$\frac{du}{dx} = 2x \qquad \frac{du}{2x} = dx$

$\int x(u^{-1/3})\frac{du}{2x}$

$\frac{1}{2}\int u^{-1/3}\,du$

$\frac{1}{2}\left(\frac{u^{2/3}}{2/3}\right)+C$

$\frac{3}{4}(u^{2/3})+C$

$= \frac{3}{4}(x^2+1)^{2/3}+C$

(15)
$\int x^{-2/3}(x^{1/3}+1)^{1/2}\,dx$

$u = x^{1/3}+1 \qquad \frac{du}{dx} = \frac{1}{3}x^{-2/3}$

$\frac{du}{1/3x^{-2/3}} = dx$

$3x^{2/3}\,du = dx$

$\int x^{-2/3}(u)^{1/2}\,3x^{2/3}\,du$

$3\int u^{1/2}\,du$

$3\left(\frac{u^{3/2}}{3/2}\right)+c$

$2(u^{3/2})+c$

$=2(x^{1/3}+1)^{3/2}+c$

(17)
$\int \frac{1}{x}(\underbrace{\ln x}_{u})^{1/2}\,dx$

$\frac{du}{dx} = 1/x$

$x\,du = dx$

$\int \frac{1}{x}(u^{1/2})\,x\,du$

$\int u^{1/2}\,du$

$\frac{u^{3/2}}{3/2}+c$

$\frac{2}{3}u^{3/2}+c$

$=\frac{2}{3}(\ln x)^{3/2}+c$

(19)
$\int e^{\overset{u}{1-x}}\,dx$

$u = 1-x \qquad \frac{du}{dx} = -1$

$-du = dx$

$-\int e^u\,du$

$= -e^{(1-x)}+c$

(21)
$\int xe^{\overset{u}{1-x^2}}\,dx$

$\frac{du}{dx} = -2x \qquad \frac{du}{-2x} = dx$

$\int xe^u\left(\frac{du}{-2x}\right)$

$-\frac{1}{2}\int e^u\,du$

$-\frac{1}{2}e^u+c$

$=\frac{1}{2}e^{1-x^2}+c$

(23)
$\int x^{-1/2}e^{\overset{u}{x^{1/2}}}\,dx$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$2x^{1/2}\,du = dx$

$\int x^{-1/2}e^u\,(2x^{\frac{1}{2}}\,du)$

$2\int e^u\,du$

$=2e^{x^{1/2}}+c$

(25)
$\int (\underbrace{3x+5}_{u})^{-1}\,dx$

$\frac{du}{dx} = 3 \qquad \frac{du}{3} = dx$

$\int u^{-1}\,\frac{du}{3}$

$\frac{1}{3}\int u^{-1}\,du$

$=\frac{1}{3}\ln u+C$

$=\frac{1}{3}\ln(3x+5)+C$

(27)
$\int x(\underbrace{x^2+3}_{u})^{-4}\,dx$

$\frac{du}{dx} = 2x \qquad \frac{du}{2x} = dx$

$\int x\,(u)^{-4}\,\frac{du}{2x}$

$\frac{1}{2}\int u^{-4}\,du$

$-\frac{1}{2}\left(\frac{u^{-3}}{-3}\right)+c$

$\frac{1}{6}\left(\frac{1}{(x^2+3)^3}\right)+C$

(29)
$\int \frac{e^{-x}}{e^{-x}+1}\,dx$

$\int e^{-x}(\underbrace{e^{-x}+1}_{u})^{-1}\,dx$

$\frac{du}{dx} = -1e^{-x}$

$\frac{du}{-e^{-x}} = dx$

$\int e^{-x}(u)^{-1}\,\frac{du}{-e^{-x}}$

$-\int u^{-1}\,du$

$= -\ln(e^{-x}+1)+C$

(31)
$\int \frac{e^{2x}-e^{-2x}}{\underbrace{e^{2x}+e^{-2x}}_{u}}\,dx$

$\frac{du}{dx} = 2e^{2x}-2e^{-2x}$

$\frac{du}{2(e^{2x}-e^{-2x})} = dx$

$\int \frac{e^{2x}-e^{-2x}}{u}\;\frac{du}{2(e^{2x}-e^{-2x})}$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln u+c$

$=\frac{1}{2}\ln(e^{2x}+e^{-2x})+c$

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$\int \frac{1}{x \ln x^2}\, dx$

$u = \ln x^2$

$\frac{du}{dx} = \frac{2x}{x^2} = \frac{2}{x}$

$\frac{x}{2} du = dx$

$\int \frac{1}{xu} \frac{x}{2} du$

$\frac{1}{2} \int \frac{1}{u} du$

$\frac{1}{2} \ln u + C$

$= \frac{1}{2} \ln(\ln x^2) + C$

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$p(x) = \int p'(x)\, dx$

$= \int \frac{-24x}{\underbrace{(3x^2+1)}_{u}{}^2}\, dx$

$\frac{du}{dx} = 6x \qquad \frac{du}{6x} = dx$

$\int \frac{-24x}{u^2} \frac{du}{6x} = \int -4u^{-2} du$

$-4\left(\frac{u^{-1}}{-1}\right) + C$

$= 4(3x^2+1)^{-1} + C$

$= \frac{4}{(3x^2+1)} + C$

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$R(x) = \int R'(x)\, dx$

$R(x) = \int 4x\, (\underbrace{10 - x^2}_{u})\, dx$

$\frac{du}{dx} = -2x \qquad \frac{du}{-2x} = dx$

$\int 4x\, (u) \frac{du}{-2x}$

$-2 \int u\, du$

$-2 \frac{u^2}{2} + C$

$= -(10 - x^2)^2 + C$

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$\int (\underbrace{x^5 + x^4 + 1}_{u})^2 (5x^3 + 4x^2)\, dx$

$\frac{du}{dx} = 5x^4 + 4x^3$

$\frac{du}{5x^4 + 4x^3} = dx$

$\int u^2 (5x^3 + 4x^2) \frac{du}{5x^4 + 4x^3}$

$\int u^2 (\cancel{5x^3 + 4x^2}) \frac{du}{x\,(\cancel{5x^3 + 4x^2})}$

$\int u^2 \frac{du}{x}$ ← still an x ①

can't solve for x using u. ②