Section 5.2 The Second Derivative p. 314

HW p. 323 (1-11) ODD ; (21-35) ODD, 43

① $f(x) = 4x^5 + x^2 + x + 3$; $\mathbb{R}$

$f'(x) = 20x^4 + 2x + 1$

$f''(x) = 80x^3 + 2$

③ $f(x) = \frac{1}{(x+1)}$; $x \neq -1$

$f'(x) = (-1)(x+1)^{-2}(1)$

$f''(x) = 2(x+1)^{-3}(1) = \frac{2}{(x+1)^3}$

$x \neq -1$

⑤ $f(x) = \sqrt{2x+1}$; $x \geq -1/2$

$f'(x) = \frac{1}{2}(2x+1)^{-1/2}(2)$

$f''(x) = \frac{-1}{2}(2x+1)^{-3/2}(2)$

$= \frac{-1}{(2x+1)^{3/2}}$; $x \neq -\frac{1}{2}$

⑦ $f(x) = 4x^5 + x^2 + x + 3$

$f'''(x) = 240x^2$

$f^{iv}(x) = 480x$

⑨ $f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -1x^{-2}$

$f''(x) = 2x^{-3}$

$f'''(x) = -6x^{-4}$

$f^{iv}(x) = 24x^{-5}$

⑪ $f(x) = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3}$

$f''(x) = \frac{-2}{9}x^{-4/3}$

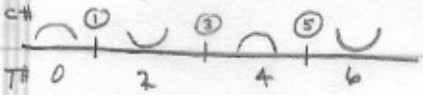
$f'''(x) = \frac{8}{27}x^{-7/3}$

$f^{iv}(x) = \frac{-56}{81}x^{-10/3}$

㉑ $f''(x) = (x-1)(x-3)(x-5)$; $\mathbb{R}$

$x-1=0 \quad x-3=0 \quad x-5=0$

C# : $x=1, \; x=3, \; x=5$



$f''(0) = (-)(-)(-)$ ∩

$f''(2) = (+)(-)(-)$ ∪

$f''(4) = (+)(+)(-)$ ∩

$f''(6) = (+)(+)(+)$ ∪

Points of Inf. : $(1, f(1)), (3, f(3)), (5, f(5))$

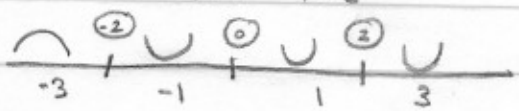
C. UP : $(1, 3), (5, \infty)$

C. Down : $(-\infty, 1), (3, 5)$

㉓ $f''(x) = (x+2)(x^2)(x-2)^2$; $\mathbb{R}$

$x+2=0 \quad x^2=0 \quad x-2=0$

C# : $x=-2 \quad x=0 \quad x=2$



$f''(-3) = (-)(+)(+)$ ∩

$f''(-1) = (+)(+)(+)$ ∪

$f''(1) = (+)(+)(+)$ ∪

$f''(3) = (+)(+)(+)$ ∪

Points of Inflection :

$(-2, f(-2))$

C. UP : $(-2, +\infty)$

C. Down : $(-\infty, -2)$

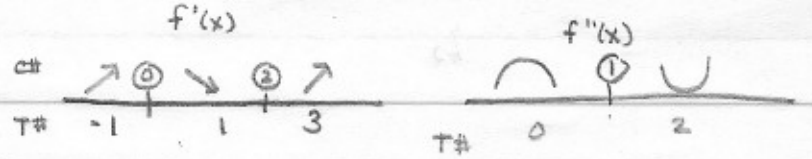
(25) $f(x) = x^3 - 3x^2 + 2$; $\mathbb{R}$

$f'(x) = 3x^2 - 6x = 3x(x-2)$; $\mathbb{R}$

$f''(x) = 6x - 6 = 6(x-1)$; $\mathbb{R}$

$f'(x) = 0$ $\quad$ $f''(x) = 0$

$x = 0, x = 2$ $\quad$ $x = 1$



$f'(-1) = (-)(-) = \oplus$ $\quad$ $f'(1) = (+)(-) = \ominus$ $\quad$ $f''(0) = \ominus \cap$ $\quad$ $f''(2) = \oplus \cup$

$f'(3) = (+)(+) = \oplus$

INC : $(-\infty, 0), (2, \infty)$ $\quad$ R. Max : $(0, f(0)) = (0, 2)$ $\quad$ C. UP : $(1, \infty)$ $\quad$ P.O.I : $(1, f(1))$

DEC : $(0, 2)$ $\quad$ R. Min : $(2, f(2)) = (2, \ )$ $\quad$ C. DN : $(-\infty, 1)$ $\quad$ $= (1, 0)$

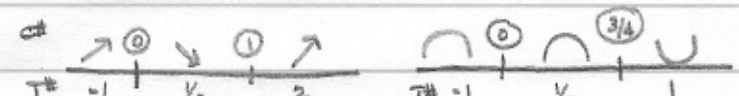
(27) $f(x) = 4x^5 - 5x^4 + 1$; $\mathbb{R}$

$f'(x) = 20x^4 - 20x^3 = 20x^3(x-1)$; $\mathbb{R}$

$f''(x) = 80x^3 - 60x^2 = 20x^2(4x-3)$; $\mathbb{R}$

$f'(x) = 0$ $\quad$ $f''(x) = 0$

$x = 0, x = 1$ $\quad$ $x = 0, x = 3/4$



$f'(-1) = (-)(-) = \oplus$ $\quad$ $f''(-1) = (+)(-) = \ominus$

$f'(1/2) = (+)(-) = \ominus$ $\quad$ $f''(1/2) = (+)(-) = \ominus$

$f'(2) = (+)(+) = \oplus$ $\quad$ $f''(1) = (+)(+) = \oplus$

INC : $(-\infty, 0), (1, \infty)$ $\quad$ R. Max : $(0, f(0)) = (0, 1)$ $\quad$ C. UP : $(3/4, \infty)$ $\quad$ P.O.I : $(3/4, f(3/4))$

DEC : $(0, 1)$ $\quad$ R. Min : $(1, f(1)) = (1, 0)$ $\quad$ C. DN : $(-\infty, 3/4)$

(29) $f(x) = x^4 + 6x^2 - 2$; $\mathbb{R}$

$f'(x) = 4x^3 + 12x = 4x(x^2 + 3)$; $\mathbb{R}$

$f''(x) = 12x^2 + 12 = 12(x^2 + 1)$; $\mathbb{R}$

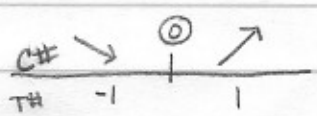
$f'(x) = 0$ $\quad$ $f''(x) = 0$

$4x(x^2+3) = 0$ $\quad$ $12(x^2+1) = 0$

$4x = 0$ $\quad$ $x^2 + 3 > 0$ $\quad$ $12 \neq 0$ $\quad$ $x^2 + 1 > 0$

FACT $\quad$ FACT $\quad$ Fact

$x = 0$ $\quad$ *Always $\cup$



$f''(-1) = (-)(+) = \ominus$

$f''(1) = (+)(+) = \oplus$

INC : $(0, \infty)$ $\quad$ R. Max : None $\quad$ C. UP : $(-\infty, \infty)$ $\quad$ P. O. I. : None

DEC : $(-\infty, 0)$ $\quad$ R. Min : $(0, f(0)) = (0, -2)$ $\quad$ C. DN : Nowhere

31. $f(x) = 5 + 8x^3 - 3x^4$; $\mathbb{R}$

$f'(x) = 24x^2 - 12x^3 = 12x^2(2-x)$; $\mathbb{R}$

$f''(x) = 48x - 36x^2 = 12x(4-3x)$; $\mathbb{R}$

$f'(x) = 0$ $\quad$ $f''(x) = 0$

$x = 0 \quad x = 2$; $\quad x = 0 \quad x = -4/3$

INC: $(-\infty, 2)$ $\quad$ C.UP: $(0, \infty)$ $\quad$ P.O.I: $(0, f(0)) = (0, 5)$ $\quad$ R. Max: $(2, f(2)) = (2, \ )$

DEC: $(2, \infty)$ $\quad$ C.DN: $(-\infty, 0)$ $\quad$ R. Min: —

C#: ↗ (0) ↗ (2) ↘ $\quad$ T#: -1, 1, 3

$f'(-1) = (+)(+) = +$ $\quad$ $f'(1) = (+)(+) = +$ $\quad$ $f'(3) = (+)(-) = -$

f'': ∩ $-4/3$ ∩ 0 ∪ $\quad$ -2, -1, 1

$f''(-2) = (-)(+) = -$ $\quad$ $f''(-1) = (-)(+) = -$ $\quad$ $f''(1) = (+)(+) = +$

33. $f(x) = x^2 e^x$; $\mathbb{R}$

$f'(x) = x^2 e^x + 2xe^x = xe^x(x+2)$; $\mathbb{R}$

$f''(x) = e^x(2x+2) + (x^2+2x)e^x = e^x(x^2+4x+2)$; $\mathbb{R}$

$f'(x) = 0$ $\quad$ $f''(x) = e^x(x^2+4x+2) = 0$

$xe^x(x+2) = 0$ $\quad$ $e^x > 0$; $(x^2+4x+2) = 0$

$x = 0$; $e^x > 0$; $x + 2 = 0$ $\quad$ $x = \dfrac{-b \pm \sqrt{b^2 - 2ac}}{2a}$

$x = -2$ $\quad$ $x = \dfrac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2}$

$x \approx \dfrac{-4 \pm \sqrt{8}}{2}$

$x_1 \approx -3.41 \quad x_2 \approx -0.59$

C#: ↗ (-2) ↘ (0) ↗ $\quad$ -3, -1, 1

$f'(-3) = (-)(+)(-) = +$ $\quad$ $f'(-1) = (-)(+)(+) = -$ $\quad$ $f'(1) = (+)(+)(+) = +$

f'': ∪ -3.41 ∩ -0.59 ∪ $\quad$ -4, -1, 0

$f''(-4) = (+)(2) = +$ $\quad$ $f''(-1) = (+)(-1) = -$ $\quad$ $f''(0) = (+)(2) = +$

INC: $(-\infty, -2)$; $(0, \infty)$ $\quad$ C.UP: $(-\infty, -3.41)$; $(-0.59, \infty)$ $\quad$ P.O.I: $(-3.41, 0.38)$ $\quad$ R. Max: $(-2, 0.54)$

DEC: $(-2, 0)$ $\quad$ C.DN: $(-3.41, -0.59)$ $\quad$ $(-0.59, 0.19)$ $\quad$ R. Min: $(0, 0)$

35. $f(x) = e^{-x^4}$

$f'(x) = -4x^3 e^{-x^4}$

$f''(x) = (-4x^3)(-4x^3 e^{-x^4}) + (-12x^2)(e^{-x^4})$

$= 4x^2 e^{-x^4}(4x^4 - 3)$

$f'(x) = -4x^3 e^{-x^4} = 0$; $\quad f''(x) = 4x^2 e^{-x^4}(4x^4 - 3) = 0$

$x = 0$ $\quad$ $x = 0$; $x = \pm(3/4)^{1/4}$

C#: ↘ (-1) ↘ (1) $\quad$ -1, 1

$f'(-1) = (+)(+) = +$ $\quad$ $f'(1) = (-)(+) = -$

f'': ∩ -0.931 ∪ 0 ∩ 0.931 ∪ $\quad$ -1, $-1/2$, $1/2$, 1

$f''(-1) = +1.47$ $\quad$ $f''(-1/2) = -2.58$ $\quad$ $f''(1/2) = -2.58$ $\quad$ $f''(1) = +1.47$