

Section 5.1 The First Derivative p. 295

HW p. 307; 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 22, 31, 33, 35, 45

1. Relative Maximum: $x = 27$ ^{May} ^{June} 12

Relative Minimum: $x = 1$ _{June}

3. Relative Minimum: $x = 1400, 1600, 1700, 1850, 1900$

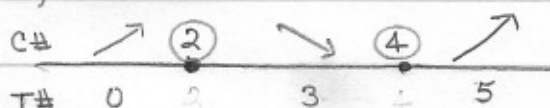
Relative Maximum: $x = 1300, 1690, 1800, 2000$

5. Relative Maximum: $x = 2500, 3250$

Relative Minimum: $x = 2750$

7. $f'(x) = p(x)(x-2)(x-4)$; $f(2) = 2$, $f(4) = -2$

CRITICAL #'s: $f'(x) = 0$



$p(x) > 0$ $x-2=0$ $x-4=0$

$$x = 2$$

$$x = 4$$

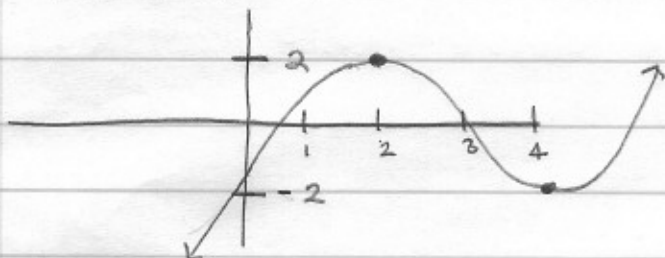
$$f'(0) = (+)(-)(-) = +$$

$$f'(5) = (+)(+)(+) = +$$

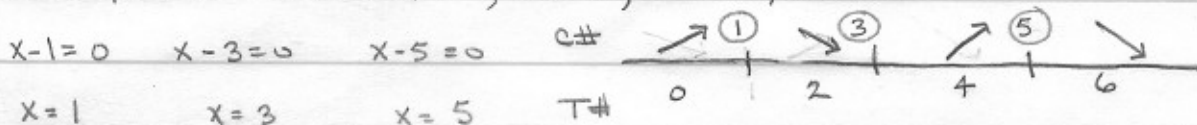
R. Min @ $f(4) = -2$

$$f'(3) = (+)(+)(-) = -$$

R. Max @ $f(2) = 2$



9. $f'(x) = p(x)(x-1)(x-3)(x-5)^2$; $f(1) = 1$, $f(3) = -2$, $f(5) = 5$



$$x-1=0$$

$$x-3=0$$

$$x-5=0$$

$$x = 1$$

$$x = 3$$

$$x = 5$$

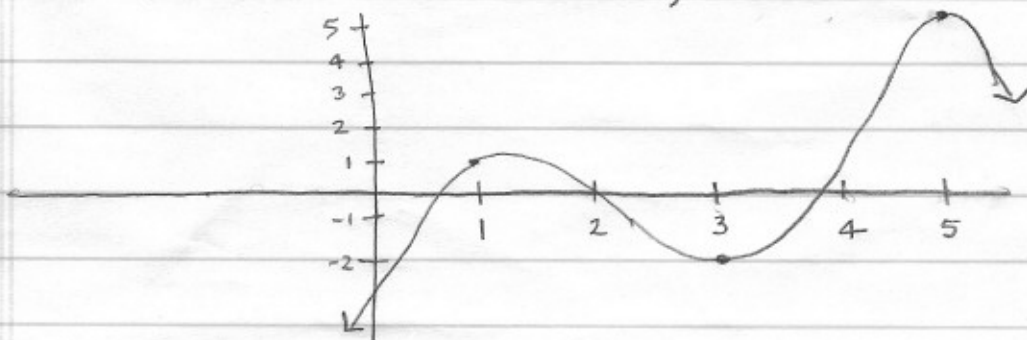
$$f'(0) = (+)(-)(-)(+) = +$$

$$f'(2) = (+)(+)(-)(+) = -$$

$$f'(4) = (+)(+)(+)(+) = +$$

$$f'(6) = (+)(+)(+)(+) = +$$

R. Min @ $f(3) = -2$ R. Max @ $f(1) = 1$, $f(5) = 5$



11. $f'(x) = p(x)(x+1)(x-1)^2(x-2)^5$; $f(-1)=1$, $f(1)=3$, $f(2)=0$

$x+1=0$

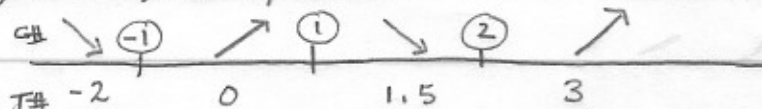
$x-1=0$

$x-2=0$

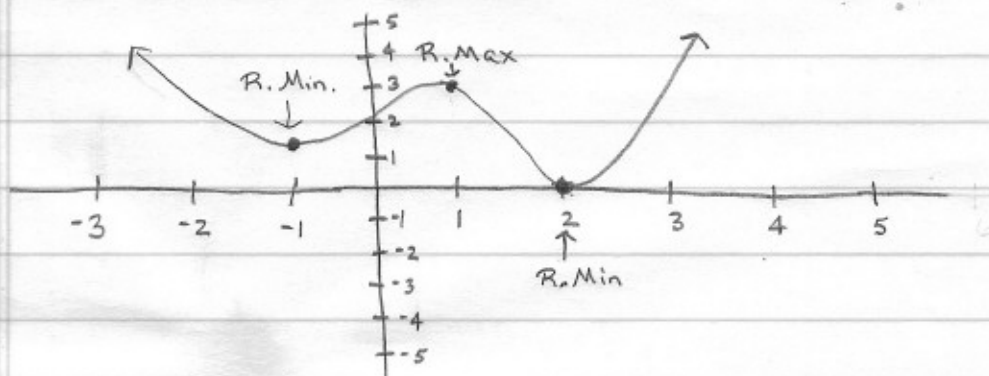
$x=-1$

$x=1$

$x=2$



$f'(-2) = (+)(-)(-)(-) = -$ $f'(0) = (+)(+)(-)(-) = +$ $f'(1.5) = (+)(+)(+)(-) = -$ $f'(3) = (+)(+)(+)(+) = +$

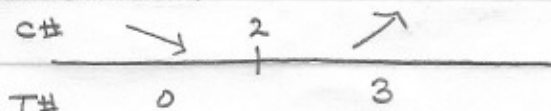


13. $f(x) = x^2 - 4x + 1$ Domain: $\mathbb{R}$

$f'(x) = 2x - 4$

C# : $2x - 4 = 0$

$x = 2$



$f'(0) = 2(0) - 4$

$f'(3) = 2(3) - 4$

$= -4 \ominus$

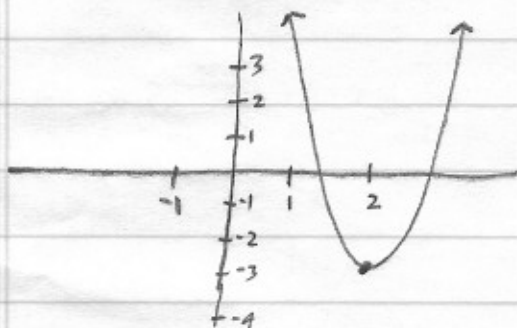
$= 2 \oplus$

{ INC : $(2, +\infty)$

{ DEC : $(-\infty, 2)$

{ R. Max : None

{ R. Min : $(2, f(2)) = (2, 2^2 - 4(2) + 1) = (2, -3)$



15. $f(x) = x^3 + 3x + 1 ; \mathbb{R}$

$f'(x) = 3x^2 + 3 = 3(x^2 + 1)$

$3(x^2 + 1) = 0$

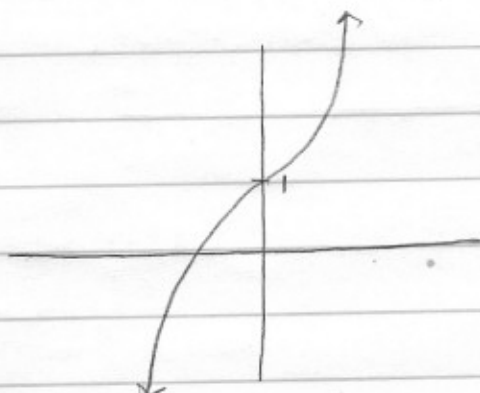
$3(x^2 + 1) > 0$ ALWAYS

Critical #s : None

R. Max./Min. : None

INC : $(-\infty, \infty)$

DEC : —



17. $f(x) = x^4 + 2x^2 + 1$

$f'(x) = 4x^3 + 4x$

$f'(x) = 4x(x^2 + 1)$

$4x = 0 \quad x^2 + 1 > 0$ (Always)

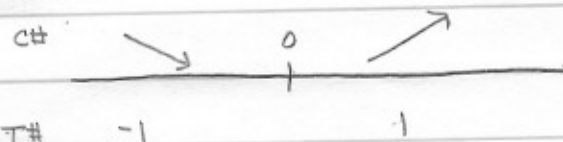
$x = 0$

Critical # : $x = 0$

R. Max : None

R. Min : $(0, f(0)) = (0, 1)$

INC : $(0, \infty)$ DEC : $(-\infty, 0)$

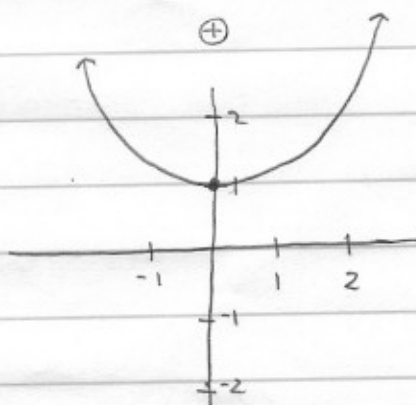


$f'(-1) = (-)(+)$

$f'(1) = (+)(+)$

$\ominus$

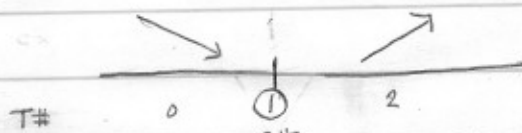
$\oplus$



19. $f(x) = 1 - (1-x)^{2/3} ; \mathbb{R}$

$f'(x) = -\frac{2}{3}(1-x)^{-1/3}(-1)$

$= \frac{2}{3(1-x)^{1/3}} ; x \neq 1$

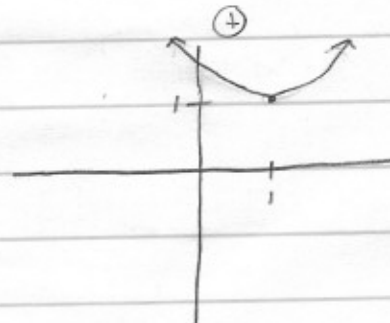


$f'(0) = \frac{(+)}{(-)}$

$f'(2) = \frac{(+)}{(+)}$

$\ominus$

$\oplus$



C# : 2 will never equal to zero (N=0)

Undefined : $1-x=0 \quad x=1$ (D=0)

R. Min : $(1, f(1)) = (1, 1)$

INC : $(1, \infty)$ DEC : $(-\infty, 1)$

$$23. f(x) = 8x^3 - 24x^2 + 18x + 6 \quad \mathbb{R}$$

$$f'(x) = 24x^2 - 48x + 18$$

$$= 6(4x^2 - 8x + 3)$$

$$6(4x^2 - 8x + 3) = 0$$

$$6(2x-3)(2x-1) = 0$$

$$2x-3=0 \quad 2x-1=0$$

$$x = \frac{3}{2} \quad x = \frac{1}{2}$$



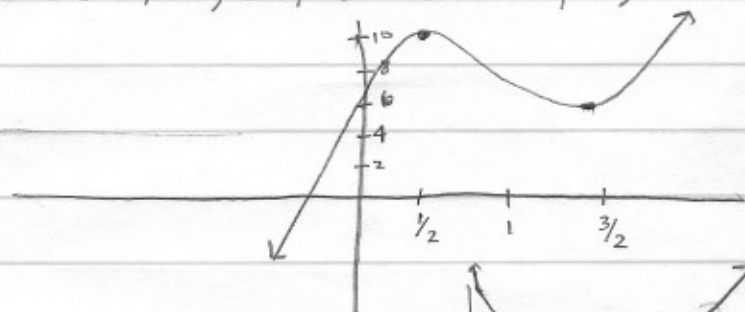
$$f'(0) = (+)(-)(-) \oplus$$

$$f'(1) = (+)(-)(+) \ominus$$

$$f'(2) = (+)(+)(+) \oplus$$

$$R. Max: (\frac{1}{2}, f(\frac{1}{2})) = (\frac{1}{2}, 10); R. Min: (\frac{3}{2}, f(\frac{3}{2})) = (\frac{3}{2}, 6)$$

$$INC: (-\infty, \frac{1}{2}), (\frac{3}{2}, \infty) \quad DEC: (\frac{1}{2}, \frac{3}{2})$$



$$31. f(x) = x - \ln x; x > 0$$

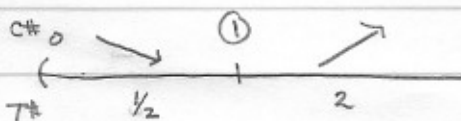
$$f'(x) = 1 - \frac{1}{x}; x \neq 0$$

$$1 - \frac{1}{x} = 0$$

$$\frac{1}{x} = 1$$

$$x = 1$$

$$f'(x) = \frac{x-1}{x}$$



$$f'(\frac{1}{2}) = \frac{(-)}{(+)} = \ominus$$

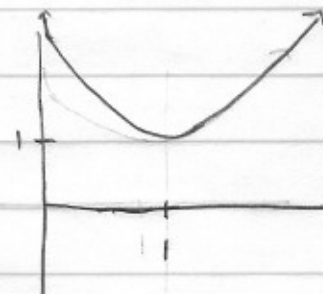
$$f'(2) = \frac{(+)}{(+)} = \oplus$$

$$R. Max: \text{None}$$

$$INC: (1, \infty)$$

$$R. Min: (1, f(1)) = (1, 1)$$

$$DEC: (-\infty, 1)$$



$$35. f(x) = x^2 e^{-x}; \mathbb{R}$$

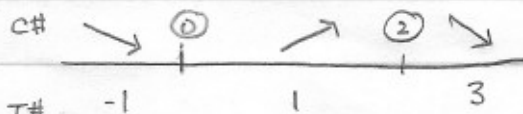
$$f'(x) = 2x e^{-x} + x^2 (-1) e^{-x}$$

$$= 2x e^{-x} - x^2 e^{-x}$$

$$= x e^{-x} (2 - x)$$

$$x e^{-x} (2 - x) = 0$$

$$x = 0 \quad \frac{e^{-x}}{\text{FACT}} > 0 \quad 2 - x = 0 \quad x = 2$$



$$f'(-1) = (-)(+)(+) \ominus$$

$$f'(1) = (+)(+)(+) \oplus$$

$$f'(3) = (+)(+)(-) \ominus$$

$$R. Max: (2, f(2)) = (2, 4e^{-2}) = (2, 0.54) \quad INC: (0, 2)$$

$$R. Min: (0, f(0)) = (0, 0) \quad DEC: (-\infty, 0); (2, +\infty)$$

