Section 5.6 Optimization and Modeling

p. 367 (1-17) ODD, 23

① Constraint: $x + y = 400$

Objective: $A = xy$ (Maximize)

$y = 400 - x$ $\quad x \geq 0$ $\quad$ [0, 400]

$A(x) = x(400 - x)$ $\quad y \geq 0 \quad 400 - x \geq 0 \quad x \leq 400$

$A(x) = 400x - x^2$; [0, 400] (x-intercepts)

$A'(x) = 400 - 2x$ $\quad A''(x) = -2$ Always $\cap$

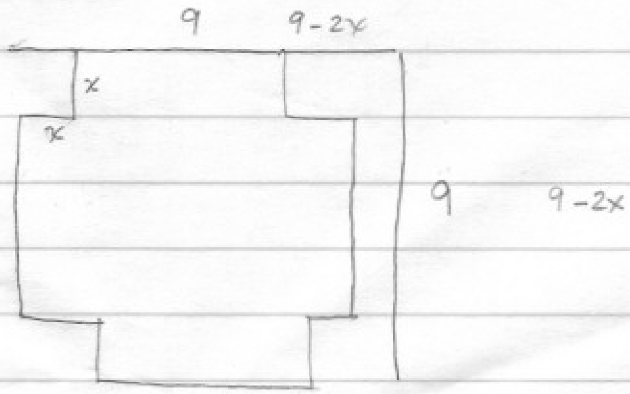
$A'(x) = 400 - 2x = 0$

$-2x = -400$ $\quad$ Dimensions: 200 x 200

$x = 200$

x	$A(x)$
0	$A(0) = (0)(400) = 0$
200	$A(200) = (200)(200) = 40{,}000$
400	$A(400) = (400)(0) = 0$

③



$9 - 2x \geq 0$

$-2x \geq -9 \quad x \leq \frac{9}{2} = 4.5$

Constraint:

Objective: $V = (9-2x)(9-2x)(x)$ $\quad (2x - 3)(2x - 9) = 0$

$V(x) = (81 - 36x + 4x^2)(x)$ $\quad x = 3/2 \quad x = 9/2$

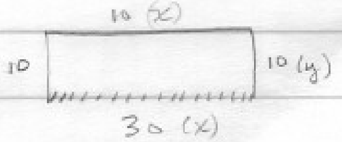
$V(x) = 81x - 36x^2 + 4x^3$ $\quad x = 1.5 \quad x = 4.5$

$V'(x) = 81 - 72x + 12x^2$ $\quad V(1.5) = 54$ *

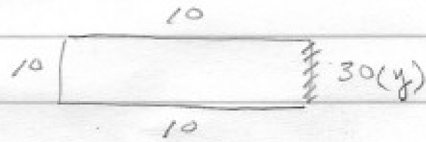
$V'(x) = 3(4x^2 - 24x + 27) = 0$ $\quad V(4.5) = 0$

$= 3(2x-3)(2x-9) = 0$

⑤ $A = 200$ sq. ft. $= xy \qquad y = \frac{200}{x} \qquad x \geq 0 \qquad y \geq 0$



OR



$$T = 10x + 30x + 10(2y) \qquad\qquad T = 10(2x) + 10y + 30y$$

$$T = 40x + 20y \qquad\qquad T = 20x + 40y$$

$$T = 40x + 20\left(\frac{200}{x}\right) \qquad\qquad T = 20x + 40\left(\frac{200}{x}\right)$$

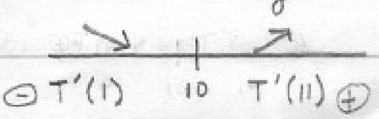
$$= 40x + 4000x^{-1} \qquad\qquad = 20x + 8000x^{-1}$$

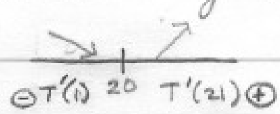
$$T' = 40 - 4000x^{-2} \qquad\qquad T' = 20 - 8000x^{-2}$$

$$40 = \frac{4000}{x^2} \qquad\qquad 20 = \frac{8000}{x^2}$$

$$x^2 = \frac{4000}{40} = 100 \qquad\qquad x^2 = \frac{8000}{20} = 400$$

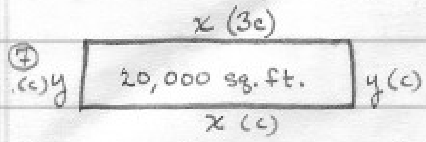
$$x = 10 \quad y = 20 \qquad\qquad x = 20 \quad y = 10$$





Dimensions: 10 ft by 20 ft

⑦



$$\left\{ \begin{array}{l} C''(x) = 80{,}000x^{-3} \\ x \geq 0 \quad C''(x) \to ⊕ \end{array} \right\}$$

$$A = xy = 20{,}000 \qquad y = \frac{20{,}000}{x} \qquad c \geq 0$$

$$C = (3c)x + cx + (c)2y$$

$$C = 4cx + 2cy$$

$$C(x) = 4cx + 2c\left(\frac{20{,}000}{x}\right)$$

$$= 4cx + 40{,}000cx^{-1} \geq 0 \qquad 4cx \geq \frac{40{,}000c}{x}$$

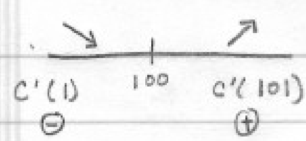
$$C'(x) = 4c - 40{,}000cx^{-2} \qquad 4cx^2 \geq 40{,}000c$$

$$= 4c - \frac{40{,}000c}{x^2} \qquad x^2 \geq 10{,}000$$

$$= \frac{4c}{x^2}(x^2 - 10{,}000) \qquad x \geq 100$$

$$x^2 - 10{,}000 = 0 \qquad x = 100 \to y = 200$$

Dimensions: 100 ft by 200 ft



9

$R = xp$ OR

$R = xp$

$= (30)(10)$

$R = (30+n)(10 - 0.2n)$

$R(n) = 300 - 6n + 10n - 0.2n^2$

$= 300 + 4n - 0.2n^2$

$R'(n) = 4 - 0.4n$ $R''(n) = -0.4$ ∩

$40 - 4n = 0$

$4n = 40$

$n = 10$

⊕   ⊖

$R'(1)$ ⊕   10 $R'(11)$ ⊖

$x = 30 + 10 = 40$ people

$R(40) = (70)(8) = \$560$

$R(x) = x\,(10 - 0.2(x - 30))$

$R(x) = x\,(10 - 0.2x + 6)$

$= 10x - 0.2x^2 + 6x$

$= -0.2x^2 + 16x$

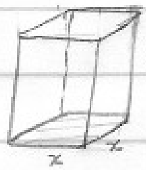
$R'(x) = -0.4x + 16$

$-0.4x = -16$

$x = 40$

$R'(1)$ 40 $R'(41)$

13.



$x>0;\ h>0$

$V = 32\,in^3 = x^2h \rightarrow h = \dfrac{32}{x^2}$

Surface area $= x^2 + 4xh$

$SA = x^2 + 4x\left(\dfrac{32}{x^2}\right)$

$= x^2 + \dfrac{144}{x} = x^2 + 128x^{-1}$

$SA' = 2x - 128x^{-2}$

$= 2x - \dfrac{128}{x^2}$

$= \dfrac{2x^3}{x^2} - \dfrac{128}{x^2}$

$= \dfrac{2x^3 - 128}{x^2}$

ZERO: $2x^3 - 128 = 0$

$2(x^3 - 64) = 0$

$x^3 = 64$

$x = (64)^{1/3} = 4$

$SA'(0)$ $\quad 4 \quad$ $SA'(5)$

$\ominus$ $\qquad\qquad$ $\oplus$

Absolute minimum when $x=4$.

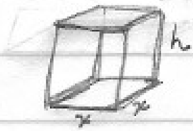
$h = \dfrac{32}{(4)^2} = 2$

Dimensions: 4" × 4" × 2"

Minimum Surface Area:

$SA_{min} = (4)^2 + 4(4)(2) = 48\,in^2$

15.



$V = 324\,in^3 = x^2h$

Cost (bottom) = 2c

Cost (sides/top) = c

$C = \underbrace{2c(x^2)}_{\text{bottom}} + \underbrace{cx^2}_{\text{top}} + \underbrace{c(4xh)}_{\text{sides}}$

$= 3cx^2 + 4cxh$

$= 3cx^2 + 4cx\left(\dfrac{324}{x^2}\right)$

$= 3cx^2 + 1296cx^{-1}$

$C' = 6cx - 1296cx^{-2}$

$= \dfrac{6cx^3 - 1296c}{x^2}$

ZERO: $6c(x^3 - 216) = 0$

$x^3 = 216$

$x = 6$

$C'(1) = -$ $\quad 6 \quad$ $C'(7) = +$

Dimensions: 6 in × 6 in × 6 in