⑨

$y = x^2 - 2x + 1$ Notice that no x-values where given.

$y = x + 1$

INTERSECTION?

$x^2 - 2x + 1 = x + 1$

$x^2 - 2x - x + 1 - 1 = 0$

$x^2 - 3x = 0$ $\longrightarrow$ Not algebraically correct:

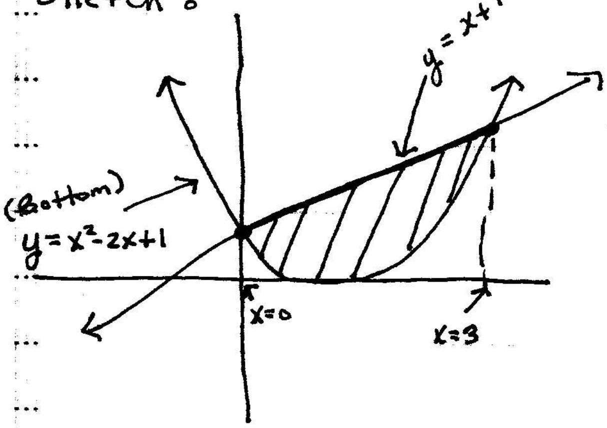
$x(x-3) = 0$ $x^2 = 3x$

$\boxed{x = 0}$ $x - 3 = 0$ $\frac{x^2}{x} = \frac{3x}{x}$

$\boxed{x = 3}$ $x = 3$ * Notice that $x=0$ was omitted.

These values of x will be used to define the interval: $[0, 3]$.

Sketch:



$$\int_0^3 [\underbrace{(x+1)}_{\text{Top}} - \underbrace{(x^2 - 2x + 1)}_{\text{Bottom}}]\, dx$$

$$\int_0^3 (x + 1 - x^2 + 2x - 1)\, dx$$

$$\int_0^3 (-x^2 + 3x)\, dx$$

$$\frac{-x^3}{3} + \frac{3x^2}{2} \Big|_{0=a}^{3=b}$$ Note: $F(x) = \frac{-x^3}{3} + \frac{3x^2}{2}$

$$= F(3) - F(0)$$

$$= \left[\frac{-(3)^3}{3} + \frac{3(3)^2}{2}\right] - \left[\frac{-(0)^3}{3} + \frac{3(0)^2}{2}\right]$$

$$= \frac{9}{2} = 4.5$$

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$y = -x^2$ $\qquad x = 0$

$y = -3x - 1$ $\qquad x = 2$

Check for a possible point of intersection within the interval $[0,2]$.

$-x^2 = -3x - 1$ $\qquad$ or $\qquad -3x - 1 = -x^2$

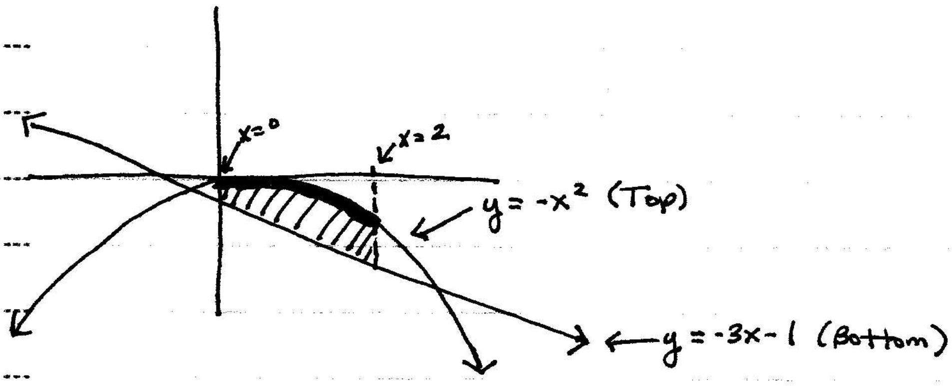
$-x^2 + 3x + 1 = 0$ $\qquad x^2 - 3x - 1 = 0$

$x^2 - 3x - 1 = 0$ (You can multiply through by -1.)

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)} = \frac{3 \pm \sqrt{13}}{2} = \begin{matrix} x_1 = 3.3028 \\ x_2 = -0.3028 \end{matrix}$$

These curves do not intersect within the interval $[0,2]$.

Sketch:



$$\int_0^2 [(-x^2) - (-3x - 1)]\, dx$$

$$\int_0^2 (-x^2 + 3x + 1)\, dx$$

$$= \frac{-x^3}{3} + \frac{3x^2}{2} + x \Big|_0^2$$

$$= \left[-\frac{(2)^3}{3} + \frac{3(2)^2}{2} + 2\right] - \left[-\frac{(0)^3}{3} + \frac{3(0)^2}{2} + 0\right] = \frac{16}{3} \text{ or } 5.33$$

(17)

$y = x^2 + 4x + 4$

$y = 4 - x$

Find the interval(s) you need by finding the point(s) of intersection.

$x^2 + 4x + 4 = 4 - x$

$x^2 + 4x + x + 4 - 4 = 0$

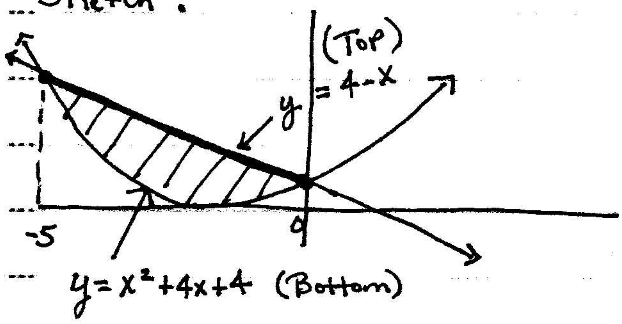
$x^2 + 5x = 0$

$x(x+5) = 0$

$\boxed{x = 0}$ $\quad x + 5 = 0$

$\boxed{x = -5}$ $\qquad$ INTERVAL: $[-5, 0]$

Sketch:



$$\int_{-5}^{0} \left[(4-x) - (x^2+4x+4)\right] dx$$

$$\int_{-5}^{0} (4 - x - x^2 - 4x - 4)\, dx$$

$$\int_{-5}^{0} (-x^2 - 5x)\, dx$$

$$= \frac{-x^3}{3} - \frac{5x^2}{2} \Bigg|_{-5}^{0}$$

$$= \left[\frac{-(0)^3}{3} - \frac{5(0)^2}{2}\right] - \left[\frac{-(-5)^3}{3} - \frac{5(-5)^2}{2}\right] = \frac{125}{6} = 20.833$$

(25)

$y = x^3$ $\qquad$ $x = -1$

$y = -x$ $\qquad$ $x = 1$

INTERVAL: $[-1, 1]$

INTERSECT ?

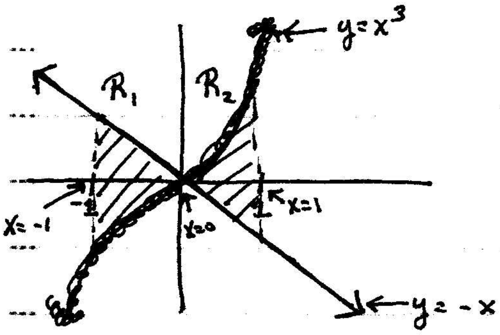
$x^3 = -x$

$x^3 + x = 0$

$x(x^2+1) = 0$

$x = 0$ * Note $x = 0$ is between $x = -1$ and $x = 1$.

Sketch:



* Notice in R_1 (region 1) that $y = -x$ is the top function over the interval $[-1, 0]$.
* Notice in R_2 (region 2) that $y = x^3$ is the top function over the interval $[0, 1]$.

$$\underbrace{\int_{-1}^{0} [(-x)-(x^3)]\,dx}_{R_1} + \underbrace{\int_{0}^{1} [(x^3)-(-x)]\,dx}_{R_2}$$

$$\int_{-1}^{0} (-x-x^3)\,dx + \int_{0}^{1} (x^3+x)\,dx$$

$$= \frac{-x^2}{2} - \frac{x^4}{4}\Big|_{-1}^{0} + \frac{x^4}{4} + \frac{x^2}{2}\Big|_{0}^{1}$$

$$= \left[\left(\frac{-(0)^3}{2} - \frac{(0)^4}{4}\right) - \left(\frac{-(-1)^2}{2} - \frac{(-1)^4}{4}\right)\right] + \left[\left(\frac{(1)^4}{4} + \frac{(1)^2}{2}\right) - \left(\frac{(0)^4}{4} + \frac{(0)^2}{2}\right)\right] = 1.5 = \frac{3}{2}$$

(25)

$y = x^3$ $\qquad\qquad$ $x = -1$

$y = -x$ $\qquad\qquad$ $x = 1$

INTERVAL: $[-1, 1]$

INTERSECT ?

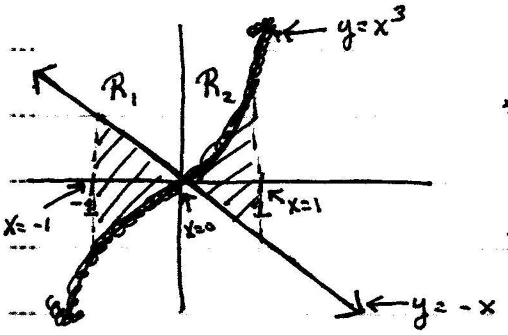
$x^3 = -x$

$x^3 + x = 0$

$x(x^2+1) = 0$

$x = 0$ $\quad$ * Note $x = 0$ is between $x = -1$ and $x = 1$.

Sketch :



* Notice in R_1 (region 1) that $y = -x$ is the top function over the interval $[-1, 0]$,

* Notice in R_2 (region 2) that $y = x^3$ is the top function over the interval $[0, 1]$.

$$\overset{R_1}{\int_{-1}^{0} [(-x)-(x^3)]\,dx} + \overset{R_2}{\int_{0}^{1} [(x^3)-(-x)]\,dx}$$

$$\int_{-1}^{0} (-x-x^3)\,dx + \int_{0}^{1} (x^3+x)\,dx$$

$$= \frac{-x^2}{2} - \frac{x^4}{4}\Big|_{-1}^{0} + \frac{x^4}{4} + \frac{x^2}{2}\Big|_{0}^{1}$$

$$= \left[\left(\frac{-(0)^2}{2} - \frac{(0)^4}{4}\right) - \left(\frac{-(-1)^2}{2} - \frac{(-1)^4}{4}\right)\right] + \left[\left(\frac{(1)^4}{4} + \frac{(1)^2}{2}\right) - \left(\frac{(0)^4}{4} + \frac{(0)^2}{2}\right)\right] = 1.5 = \frac{3}{2}$$

(7)

$y = e^{x}$ $\qquad x = -\ln 2 \approx -0.693$

$y = e^{-x}$ $\qquad x = \ln 2 \approx 0.693$

INTERSECT?

(Algebraically)

$e^{x} = e^{-x}$

$e^{x} = \frac{1}{e^{x}}$

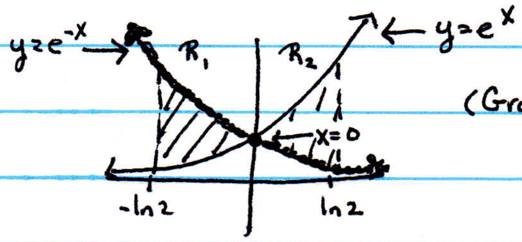
$e^{2x} = 1$

$2x = \ln 1$

$2x = 0$

$x = 0$

(Graphically)



Sketch:

$$\overbrace{\int_{-\ln 2}^{0} (e^{-x} - e^{x})\,dx}^{R_1} + \int_{0}^{\ln 2} (e^{x} - e^{-x})\,dx$$

$$\frac{e^{-x}}{-1} - e^{x} \Big|_{-\ln 2}^{0} + e^{x} - \left(\frac{e^{-x}}{-1}\right) \Big|_{0}^{\ln 2}$$

$$-e^{-x} - e^{x} \Big|_{-\ln 2}^{0} + e^{x} + e^{-x} \Big|_{0}^{\ln 2}$$

$$\left[(-e^{-(0)} - e^{0}) - (-e^{-(-\ln 2)} - e^{(-\ln 2)})\right] + \left[(e^{\ln 2} + e^{-\ln 2}) - (e^{0} - e^{-(0)})\right] =$$

$$\left[(-1-1) - (-2-\tfrac{1}{2})\right] + \left[(2+\tfrac{1}{2}) - (1-1)\right]$$

$$\left[(-2) - (\tfrac{-3}{2})\right] + \left[\tfrac{3}{2}\right]$$

$$\left[\tfrac{-1}{2}\right] + \left[\tfrac{3}{2}\right] = \tfrac{2}{2} = 1$$