TWO-FACTOR ANALYSIS OF VARIANCE

"Statistics is the art of never having to say you're wrong and never having to say you're certain." ~ Author Unknown

Intro:

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- Factorial design =

Nomenclature:

- Each level of an IV is crossed with each level of another IV
- (levels of 1st factor) X (levels of 2nd factor)
- 2 X 2 (2 by 2):
- 3 X 3 (3 by 3):
- 3 X 4 X 2 (3 by 4 by 2):

Example of a 2 X 3 Design:

Effect of heat vs. humidity on thinking/working proficiency
IV's:
1. Humidity: low to high
2. Heat: 70°, 80°, 90°
DV: problem-solving task

Create a 2-by-3 matrix using the 2 IV's:

		Factor B: Temperature		
		70° Room	80° Room	90° Room
Factor A: Humidity	Low Humidity			
	High Humidity			

Mean Differences: The 2-factor ANOVA will evaluate 3 separate sets of mean differences

1)
2)
3)

Hypothesis Testing With F-Ratio's:

F = <u>variance btn sample means (actual mean diffs in the data)</u>
variance expected by chance or sampling error
(differences expected if there were no txt effect)

Main Effects & Interactions

- Identify the IV's as factor A & factor B:
  Factor A – Humidity
  Factor B – Temperature
- Goal of the experiment:

		Factor B: Temperature			
		70° Room	80° Room	90° Room	
Factor A: Humidity	Low Humidity	M = 85	M = 80	M = 75	M =
	High Humidity	M = 75	M = 70	M = 65	M =
		M =	M =	M =	

<u>Main Effects:</u>

Main Effect of Humidity:

Main Effect of Temperature:

Hypotheses:

Factor A involves the comparison of 2 diff levels of humidity:

- H_0:

- H_1:

F_A = variance (diffs) btn the means for factor A/row means
variance (diffs) expected by chance/error

Factor B involves the comparison of 3 different temp conditions:

- H_0:

- H_1:

F_B = variance (diffs) btn means for factor B/column means
variance (diffs) expected by chance/error

Interactions:
Definitions:

New data: old data altered to depict an interaction

		Factor B: Temperature			
		70° Room	80° Room	90° Room	
Factor A: Humidity	Low Humidity	M = 80	M = 80	M = 80	M =
	High Humidity	M = 80	M = 70	M = 60	M =
		M =	M =	M =	

F = <u>variance (mean diffs) not explained by main effects</u>
variance (diffs) expected by chance/error

Null hypothesis:

Alternative hypothesis:

Reporting Interactions

1. tables

2. bar graphs

3. line graphs

Graphs and interactions:

Activity: Fasting & Meditation (Handout)

The Structure of the 2-Factor Analysis

The 2-factor ANOVA is composed of 3 distinct hypothesis tests:

1) *the main effect of factor A* (the A-effect)

2) *the main effect of factor B* (the B-effect)

3) *the interaction* (the A X B interaction)

All 3 F-ratios have the same basic structure:

$$F = \frac{\text{variance (mean differences) between treatments}}{\text{variance (differences) expected by chance/error}}$$

The Overall Analysis Is Divided Into 2 Stages:

1st stage: the total variability is divided into 2 components:
- btn-txts variability
- within-txts variability

identical to the single-factor analysis of variance with each cell in the 2-factor data matrix viewed as a separate txt condition

2nd stage: partitions the btn-txts variability into 3 components:
1. differences attributed to factor A
2. differences attributed to factor B
3. and any remaining mean diffs that define the interaction

Notation and Formulas for the 2-Factor ANOVA

- The relationship between level of arousal and performance.
- Factor A: levels of task difficulty make up the rows – 2 levels: easy & difficult
- Factor B: levels of arousal make up the columns – 3 levels: low, medium, & high

Are the mean differences observed in the data significantly greater than would be expected just by chance?

Stage 1 of the 2-Factor Analysis: separates the total variability into 2 components: between txts & within txts; each cell is treated as a separate txt condition.

Total variability: measures the variability for all N = 30 scores

$$SS_{total} = \sum X^2 - \frac{G^2}{N}$$

df_{total} = N – 1 =

Between-treatments variability: each txt condition corresponds to a cell in the matrix

$$SS_{between-treatments} = \sum \frac{T_k^{\,2}}{n_k} - \frac{G^2}{N}$$

$df_{between\ treatments}$ = number of cells – 1 =

Within-treatments variability: to compute the variance within treatments, first compute the SS and df = n – 1 for each of the individual treatment conditions

$$SS_{within-treatments} = \sum X^2 - \sum \frac{T_k^{\,2}}{n_k} =$$

$df_{within\ treatments}$ =

Note that the 2 components add to equal the total for both SS values and df values:

$SS_{between\ treatments} + SS_{within\ treatments} = SS_{total}$

$df_{between\ treatments} + df_{within\ treatments} = df_{total}$

Stage 2 of the 2-Factor Analysis:
The 2nd stage will determine the numerators for the 3 F-ratios: the between-treatments variance for factor A, factor B, and the interaction.

1. *Factor A:* the main effect for factor A evaluates the mean diff btn the levels of factor A

$$SS_{factorA} = \sum \frac{T_A^{\ 2}}{n_A} - \frac{G^2}{N}$$

df_A = number of rows – 1 =

2. *Factor B:* the main effect for factor B evaluates the mean diff btn the levels of factor B

$$SS_{factorB} = \sum \frac{T_B^{\ 2}}{n_B} - \frac{G^2}{N}$$

df_B = number of columns – 1 =

3. *The A X B interaction:* the "extra" mean differences not accounted for by the main effects of the 2 factors.

$SS_{AxB} = SS_{between\ treatments} - SS_A - SS_B$ =

$df_{AxB} = df_{between\ treatments} - df_A - df_B$ =

Mean Squares & F-Ratios for the 2-Factor Analysis

3 separate hypotheses → 3 separate F-ratios

$$MS_{\text{within treatments}} = \frac{SS_{\text{within treatments}}}{df_{\text{within treatments}}} =$$

This value will form the denominator for all 3 F-ratios.

$$MS_A = \frac{SS_A}{df_A} =$$

$$MS_B = \frac{SS_B}{df_B} =$$

$$MS_{AxB} = \frac{SS_{AxB}}{df_{AxB}} =$$

F-Ratios:

$$F_A = \frac{MS_A}{MS_{\text{w/in txts}}} =$$

$$F_B = \frac{MS_B}{MS_{\text{w/in txts}}} =$$

$$F_{AxB} = \frac{MS_{AxB}}{MS_{\text{within txts}}} =$$

F-Criticals:

Factor A:

Factor B:

Factor AxB:

Interpretation of results from a 2-factor:

Graph the results:

Assumptions for the 2-Factor ANOVA

1) Independence
2) Normality
3) Homogeneity of variance

Second Example:

Relationship between eating behavior & body weight

IVs: participant's weight (normal or obese)
participant's state of hunger (full or empty stomach)

DV: the number of crackers eaten by each participant