ONE-WAY INDEPENDENT MEASURES ANOVA'S

Introduction to Analysis of Variance

Purpose:

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e.g. academic performance (number of problems solved correctly) in 3 different temperature conditions:

Treatment 1 (50°) → M = 1
Treatment 2 (70°) → M = 4
Treatment 3 (90°) → M = 1

t-Tests vs. ANOVA's

Similarities

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Differences

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Research Design Terminology

Independent variable (IV)

Quasi-independent variable

Factor

Levels of a factor

Single Factor, Independent Measures Designs

Statistical Hypotheses for ANOVA

The Test Statistic for ANOVA

The test statistic for ANOVA is very similar to the t-statistic:

$$t = \frac{\text{difference obtained between sample means}}{\text{difference expected by chance (error)}}$$

$$F = \frac{\text{variance (differences) between sample means}}{\text{variance (differences) expected by chance (error)}}$$

F-ratio based on variance

The variance in the numerator of the F-ratio:

Set 1	Set 2
$M_1 = 20$	$M_1 = 28$
$M_2 = 30$	$M_2 = 30$
$M_3 = 35$	$M_3 = 31$
$s^2 = 58.33$	$s^2 = 2.33$

The variance in the denominator:

A large value for the F-ratio:

The Logic of ANOVA

e.g. Data from experiment examining learning performance (number of problems solved correctly) under 3 temperature conditions:

Tx 1: 50° (Sample 1)	Tx 2: 70° (Sample 2)	Tx 3: 90° (Sample 3)
0	4	1
1	3	2
3	6	2
1	3	0
0	4	0
$M =$	$M =$	$M =$
$n =$	$n =$	$n =$

Goal of ANOVA:

1st step:

Next step:

Sources of Variation

The analysis process divides the total variability into 2 basic components or sources of variation:

1. **Between-Groups/Treatments Variation (s_B^2):**

2. **Within-Groups/Treatments Variation (s_W^2):**

Overall Goal of ANOVA

There are always 2 possible explanations for the differences or variance that exists between treatments:

1) **Treatment Effect, Between-Grps/Txts Variance (s_B^2):**

2) **Chance, Within-Grps/Txts Variance (s_W^2):**

3 Primary Sources for Chance Differences:

1. Individual differences
2. Experimenter error
3. Random sampling fluctuations

The F ratio:

F =

When the treatment has no effect:

When the txt does have an effect:

ANOVA Notation & Formulas

Txt 1 50° (Sample 1)	Txt 2 70° (Sample 2)	Txt 3 90° (Sample 3)	
0	4	1	$\Sigma X^2 =$
1	3	2	$G =$
3	6	2	$N =$
1	3	0	$k =$
0	4	0	
$T_1 =$	$T_2 =$	$T_3 =$	
$n_1 =$	$n_2 =$	$n_3 =$	
$M_1 =$	$M_2 =$	$M_3 =$	
$\Sigma X^2 =$	$\Sigma X^2 =$	$\Sigma X^2 =$	
$SS_1 =$	$SS_2 =$	$SS_3 =$	

$k =$

$n =$

$N =$

$T =$

$G =$

$M =$

$SS =$

F =

Analysis of Sum of Squares:

Between-groups sum of squares $(SS_B) =$

$$SS_B = \sum_{k=1}^{K} \frac{{T_k}^2}{n_k} - \frac{G^2}{N}$$

$$SS_B = \left[\frac{\left(\sum X_1\right)^2}{n_1} + \frac{\left(\sum X_2\right)^2}{n_2} + \ldots + \frac{\left(\sum X_k\right)^2}{n_k} \right] - \left[\frac{\left(\sum X_1 + \sum X_2 + \ldots + \sum X_k\right)^2}{N_{total}} \right]$$

Within-Treatments/Groups Sum of Squares *(SS_W) =*

$$SS_W = \sum X^2 - \sum \frac{(T_k)^2}{n_k}$$

$$SS_{wg} = \left[\sum X_1^2 + \sum X_2^2 + \cdots + \sum X_k^2\right] - \left[\frac{(\sum X_1)^2}{n_1} + \frac{(\sum X_2)^2}{n_2} + \cdots + \frac{(\sum X_k)^2}{n_k}\right]$$

$SS_{\text{within}} = \Sigma SS_{\text{inside each treatment}}$

Total Sum of Squares *(SS_T) =*

$$SS_{total} = \left[\sum X_1^2 + \sum X_2^2 + \cdots + \sum X_k^2\right] - \left[\frac{\left(\sum X_1 + \sum X_2 + \cdots + \sum X_k\right)^2}{N_{total}}\right]$$

$$SS_T = \sum X^2 - \frac{\left(\sum X\right)^2}{N} = \sum X^2 - \frac{G^2}{N}$$

<u>Analysis of Degrees of Freedom (df)</u>

Total degrees of freedom

$df_{\text{total}} = N - 1 =$

$df_{\text{total}} = df_{\text{within}} + df_{\text{between}} =$

Within-treatments degrees of freedom

$df_{\text{within}} = \Sigma(n - 1) = \Sigma df_{\text{in each treatment}} = N - k =$

Between-treatments degrees of freedom

$df_{\text{between}} =$ # of levels of the IV – 1 = $k - 1 =$

Calculations of Variances

Variance is defined as the mean of squared deviations: $MS = s^2 = \frac{SS}{df}$

Between-treatments mean square

$$MS_{between} = s^2_{between} = \frac{SS_{between}}{df_{between}} = \frac{SS_{between}}{k-1} =$$

Within-treatments mean square

$$MS_{within} = s^2_{within} = \frac{SS_{within}}{df_{within}} = \frac{SS_{within}}{N-k} =$$

F-ratio

$$F = \frac{MS_B}{MS_W} =$$

Summary ANOVA Table:

Source of Variation	Sum of Squares	Degrees of Freedom	Variance Estimate (Mean Square)	F-ratio
Between	$SS_B = \sum_{k=1}^{K} \frac{T_k^2}{n_k} - \frac{G^2}{N}$	K – 1	$MS_B = \frac{SS_B}{K-1}$	$F = \frac{MS_B}{MS_W}$
Within	$SS_W = \sum X^2 - \sum \frac{(T_k)^2}{n_k}$	N – K	$MS_W = \frac{SS_W}{N-K}$	
Total	$SS_T = SS_W + SS_B$	N – 1		

The *F*-Distribution