

CONFIDENCE INTERVAL FOR THE MEAN

Definition:

Sampling Error

Constructing a Confidence Interval for the Mean of a Population:

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Level of Confidence:

— **95% CI**

— **99% CI**

Formula for the Construction of a Confidence Interval for a Population with a Known Standard Deviation:

CI = sample mean $\pm$ Z * standard error

$$95\% \text{ CI} = \bar{X} \pm Z_{.05} \sigma_{\bar{X}}$$

$$99\% \text{ CI} = \bar{X} \pm Z_{.01} \sigma_{\bar{X}}$$

*Use Z values of 1.96 for a 95% CI & 2.58 for a 99% CI

EXAMPLE:

A researcher knows that the national mean on the GMAT is 500 and the standard deviation is 100. A sample of 121 seniors from an affluent private university obtains a mean of 519.

Construct a 95% confidence interval around the sample mean.

Would the national mean of 500 be included in the interval?

Construct a 99% confidence interval around the sample mean.

Would the national mean of 500 be included in the interval?

What a confidence interval tells us:

The size of the confidence interval:

Precision vs. accuracy

Practice Problems:

$$\bar{X} = 50 \quad \sigma = 8 \quad n = 100$$

95% CI:

99% CI:

$$\bar{X} = 50 \quad \sigma = 8 \quad n = 400$$

95% CI:

99% CI:

$$\bar{X} = 85 \quad \sigma = 16 \quad n = 25$$

95% CI:

99% CI:

$$\bar{X} = 85 \quad \sigma = 16 \quad n = 225$$

95% CI:

99% CI:

Formula for the Construction of a Confidence Interval for a Population with a Unknown Standard Deviation:

$$95\% \text{ CI} = \bar{X} \pm t_{.05} s_{\bar{X}}$$

$$99\% \text{ CI} = \bar{X} \pm t_{.01} s_{\bar{X}}$$

$$s_{\bar{X}} = \frac{s}{\sqrt{n}}$$

t-values → Look up in table in appendix (Appendices B & C).

Note: We use t-values because we don't know either μ (the population mean) or σ (the population standard deviation) – we have to estimate their values with $\bar{x}$ and $s_{\bar{x}}$ respectively.

Example:

In a study to determine the amount of REM sleep engaged in by young adults, 35 volunteers were monitored in a sleep lab over a 1-week period. The average amount of REM sleep per night was 96 minutes, with a standard deviation of 22 minutes.

What is the 95% confidence interval?

What is the 99% confidence interval?

Example:

A random sample of 10 male college students who had tried out for the intercollegiate basketball team had a mean height of 72" and a standard deviation of 1.4142". The population mean for adult males is 69". Construct a 95% CI and a 99% CI. Is the national mean included in either interval?

Practice Problems:

- *Given a mean of 100, a standard deviation of 12, and $n = 16$, construct a 95% confidence interval for the mean.*
- *Given a mean of 54, a standard deviation of 15, and $n = 25$, construct a 99% confidence interval for the mean.*
- *Given a mean of 6500, a standard deviation of 240, and $n = 16$, construct a 95% confidence interval for the mean.*