The Normal Curve

"I'm not an outlier; I just haven't found my distribution yet." – author unknown

Why Is the Normal Distribution Important?

What is the Normal Distribution?

When the normal distribution is graphed, it is referred to as the normal curve.

Characteristics Of The Normal Curve:

1. The curve is bell-shaped and symmetrical.
2. The mean, median, and mode are all equal.
3. The highest frequency is in the middle of the curve.
4. The frequency gradually tapers off as the scores approach the ends of the curve.
5. The curves approaches, but never meets, the abscissa, at both high and low ends.

Two parameters define the exact shape of the distribution:

Determining & Interpreting Proportions/Percentages of Scores in Areas Under the Normal Curve

Entire area under the curve:

Connection between z-scores and proportions of area under the curve:

A certain percentage of scores falls in each section under the curve:

When determining areas between z scores and computing percentiles always follow the consistent procedure below for solving the problems:

1) sketch the distribution
2) locate the mean and standard deviations
3) draw a vertical line for the score in question
4) shade in the appropriate area

The Normal Curve Table

Handout: Table A.1

The normal curve table lists the relationship between *z*-score locations and proportions in a normal distribution.

First column (A) – lists z score values corresponding to different positions in the normal distribution
Second column (B) – the proportion of the distribution between the mean and the specific score
Third column (C) – the proportion of the distribution in the tail

Proportions on the left-hand side vs. proportions on the right-hand side:

Signs for z-scores and proportions:

Finding Proportions & Probabilities for Specific *z*-score Values

1] What proportion of the normal distribution corresponds to z-score values greater than $z = +1.00$?

2] For a normal distribution, what is the probability of selecting a z-score value less than $z = +1.50$? $p(z < +1.50) = ?$

3] What proportion of the normal distribution corresponds to the tail beyond *z* = -0.50?

Finding the Z-Score Location That Corresponds to Specific Proportions

Beginning with a known proportion and looking up the corresponding *z*-scores.

1] For a normal distribution, what z-score separates the top 10% from the remainder of the distribution?

2] For a normal distribution, what z-score values separate the middle 60% of the distribution from the rest of the scores?

Practice Problems:

Find the proportion of the normal distribution that is associated with the following sections of a graph:

a. $z < +1.00$

b. $z > +0.80$

c. $z < -2.00$

d. $z > -0.33$

e. $z > -0.50$

For a normal distribution, find the z-score location that separates the distribution as follows:

a. the highest 30% from the rest of the distribution

b. the lowest 40% from the rest of the distribution

c. the highest 75% from the rest of the distribution

Probabilities/Proportions for Scores (X-Values) from a Normal Distribution

Finding probabilities for specific X values

1] It is known that IQ scores form a normal distribution with $\mu = 100$ and $\sigma = 15$, given this information, what is the probability of randomly selecting an individual with an IQ score less than 130? $p(X < 130) = ?$ What is the proportion of IQ scores that corresponds to scores less than 130?

2] Adult heights form a normal distribution with a M of 68 inches and a SD of 6 inches. What is the probability of selecting an individual from this population who is taller than 6 feet 8 inches (X = 80 inches)? $p(X > 80) = ?$

3] Scores on the SAT form a normal distribution with $\mu = 500$ and $\sigma = 100$. What is the minimum score necessary to be in the top 15% of the SAT distribution?

Finding Proportions & Probabilities Located Between Two Scores

The probability of selecting a score that is located between 2 specific values.

1] The highway department conducted a study measuring driving speeds on a local section of the interstate highway. They found an average speed of μ = 58mph with a standard deviation of σ = 10mph. The distribution was approximately normal. Given this information, what proportion of the cars was traveling between 55 and 65 mph? $p(55 < X < 65) = ?$

Practice problems:

For a normal distribution with a mean of 80 and a standard deviation of 10, find each probability value requested:

a. $p(X > 85) = ?$

b. $p(X < 95) = ?$

c. $p(X > 70) = ?$

d. $p\ (75 < X < 100) = ?$

For a normal distribution with a mean of 100 and a standard deviation of 20, find each value requested.

a. What score separates the top 40% from the bottom 60% of the distribution?

b. What is the minimum score needed to be in the top 5% of the distribution?

c. What scores form the boundaries for the middle 60% of the distribution?

Percentiles

Percentile rank:

Percentile:

Possible to rephrase the probability problems:

What is the probability of randomly selecting an individual with an IQ score less than 130? → What is the percentile rank for an IQ score or 130?

What is the minimum score necessary to be in the top 15% on the SAT? → What is the 85^{th} percentile for the distribution of SAT scores?