PROBABILITY THEORY

"Did you hear the one about the statistician? Probably...."

Probability

Definition:

Examples:

Probability may be expressed as:

- a fraction
- a proportion
- a percentage

Probability is measured from 0-1:

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-
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Equation to calculate the probability of any particular outcome:
In a situation where several different outcomes (A, B, C, D, etc.) are possible, the probability for any particular outcome is represented by the following equation:

$$\text{Probability of A} = \frac{\text{number of outcomes classified as A}}{\text{total number of possible outcomes}}$$

Examples:

Probability of selecting the king of hearts:

Probability of selecting an ace:

p(spade) =

p(heads) =

You have just bought a ticket for the local lottery and your friend has bought 2 tickets. If 500 tickets were sold:

What is the probability that you will win?

What is the probability that your friend will win?

Addition Rule of Probability/Addition Theorem of Probability/The "Or Rule" of Probability

Used:

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Equation:

- p (A or B) = p (A) + p (B)
- For mutually exclusive random events, the probability of either 1 event or another event occurring equals the sum of the probabilities of the individual events.

Examples:

p (jack or queen)

You have just bought a ticket for the local lottery and your friend has bought 2 tickets. If 500 tickets were sold, what is the probability that either you or your friend will win?

Equation for events that aren't mutually exclusive:

p (A or B) = p(A) + p(B) – p(A and B)

Example:

What is the prob of obtaining a jack or heart from a deck of cards?

Multiplication Rule of Probability/Multiplication Theorem of Probability/The "And Rule" of Probability

Used:

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Examples:

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Equation:

- p (A, B) = p(A) x p(B)
  p (A, B, C) = p(A) x p(B) x p(C)
- The probability of two or more independent events occurring together is equal to the product of their individual probabilities.

Examples:

p(heart and jack) =

If you take a 20-question T/F test that you haven't studied for, assuming random guessing, what is the probability that you will miss all 20 questions?

Concept Review

Nielson/NetRatings (2000) study of the characteristics of internet users:

Category	Frequency
Internet users who are female	30.30 million
Internet users who are male	40.20 million
Internet users who are full-time students	9.52 million
Total number of internet users	70.50 million

p(male) =

p(male internet user or female internet user) =

p(female internet user or full-time college student internet user) =

p(female internet user and full-time college student internet user) =

Conditional Probability

Definition:

Used:
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Equation for the prob of a sequence of non-independent events:
p (A, B) = *p*(A) x *p*(B|A)
p (A, B, C) = *p*(A) x *p*(B|A) x *p*(C|A, B)

Examples:
What is the probability of obtaining a straight royal flush (10, jack, queen, king, ace) in diamonds in 5 consecutive draws w/out replacement?

What is the probability of drawing aces in 4 consecutive draws w/out replacement?

What is the probability of obtaining 3 heads in 5 flips of a fair coin?

Practice Problem: What is the probability of obtaining 4 heads in 5 flips of a fair coin?

Conditional probability

The formula for the probability of B given A: $p(B|A) = \frac{p(A, B)}{p(A)}$

Example: In a study of psychotherapy outcomes for clients with and without a college degree, the results are summarized in the following table of probabilities:

		College Degree (A)		
		Yes	No	Total
Psychotherapy Outcome (B)	Good	.30	.12	.42
	Poor	.15	.43	.58
	Total	.45	.55	1.00

What is the probability that a person will have success in psychotherapy?

Given that a person is a college graduate, what is the probability that the person will have a good psychotherapy outcome?

Does the added information about education aid in predicting psychotherapy success?

PROBABILITY PRACTICE PROBLEMS

There are 900 Psychology majors and 500 Sociology majors at a university with 12,000:

1. What is the probability that the next person who comes through the student union front door will be a Psychology major?

2. What is the probability that the next person who comes through the student union front door will be a Sociology major?

3. What is the probability that the next person who comes through the student union front door will be a Psychology major or a Sociology major?

4. If 50 students are Psychology/Sociology double majors, what is the probability that the next person who comes through the student union front door will be a Psychology major or a Sociology major?

Use the following probabilities to solve the next few problems:

	Probability
Having schizophrenia	.01
Employed by the government	.25
Employed in the private sector	.65
Unemployed	.10
High sensation seeker	.20
Low sensation seeker	.10

5. What is the probability of being either employed by the government or unemployed?

6. What is the probability of being either a high sensation seeker or a low sensation seeker?

7. What is the probability of being either employed in the private sector or employed by the government?

8. What is the probability of both being employed by the government and having schizophrenia?

9. What is the probability of both being employed by the private sector and having schizophrenia?

10. What is the probability of being employed by the government, a high sensation seeker, and having schizophrenia?

11. What is the probability of being a low sensation seeker, unemployed, and having schizophrenia?

A researcher conducted a survey that asked parents to estimate the daily time 4-year-old children spent doing various activities. Use the results of the survey to solve the following several problems.

Sample Characteristics	**Frequency**	**Activities**	**Minutes Per Day**
Sex:		Chores	32
Girls	115	Educational Activities	78
Boys	135	Outside Play	65
TOTAL	250	Personal Care	60
		Sleep	510
		Socializing	45
Ethnicity:		Television: Educational	66
White	95	Television: Noneducational	294
African American	89	Videos (entertainment)	65
Latino/Hispanic	38	Video Games	125
Other	28	Other	100
TOTAL	250	TOTAL	1440

12. What is the probability of a child being male?

13. What is the probability of a child being either White or African American?

14. What is the probability of a child being female and White?

15. Disregarding the time of day, what is the overall probability that a child is watching either noneducational or educational television?

16. Disregarding the time of day, what is the overall probability that a male child is watching noneducational television?

17. Disregarding the time of day, what is the overall probability that a child is either socializing or doing chores?

18. Disregarding the time of day, what is the probability of child's being male and either watching television or playing video games?

19. Disregarding the time of day, what is the probability of a child's being female and either socializing or playing video games?

20. Disregarding the time of day, what is the probability that a child in this study is male and either White or Latino/Hispanic and sleeping?

Connection between Probability and Sampling

- Inferential statistics:
- Two stage process:

1) Identify the types of samples that probably would be obtained from a specific population

   e.g. Populations are 2 jars of marbles:
   Jar 1 – 50 white marbles & 50 black marbles
   Jar 2 – 10 white & 90 black marbles
   What color are you likely to draw if you select a single marble from jar 1? Jar 2?

2) Use the sample to make an inference about the population

   e.g. If you select a sample of n=4 marbles and all are black, which population did the sample probably come from?

- Inferential procedures are typically built around the concept of probability