MEASURES OF VARIABILITY

"An approximate answer to the right question is worth a good deal more than the exact answer to an approximate question." John Tukey

Variability:

Purposes of Measures of Variability:

1.

2.

3.

4.

Importance of a Conceptual Understanding of Variability:

1.

2.

3.

3 COMMON MEASURES OF DISPERSION

1. Range

2. Variance

3. Standard Deviation

RANGE

Definitions:

1.

2.

Formula:

Range = URL X_{max} – LRL X_{min}

Example:
Calculate the range for the following data: 3, 7, 12, 8, 5, 10
X_{max} =
X_{min} =
Range =

Advantages and Disadvantages of the Range:

-
-
-

Class A: 13, 23, 33, 43, 53, 63, 73, 83, 93, 100
Class B: 13, 85, 85, 86, 87, 87, 88, 88, 89, 100

Example:
Find the range for the following data:
3, 4, 5, 7, 9, 10, 11, 13
Range = URL X_{max} *– LRL* X_{min} *=*

CLASS EXERCISE

Consider the 2 distributions on the next page:

- Calculate the means, medians, modes, and ranges for each.
- How are the 2 samples different?
- Which measure of central tendency is the most representative for each sample?
- Is the range an accurate measure of variability for each of the samples? Why or why not?

Two Samples

Class A	**Class B**
1	**1**
5	**1**
5	**1**
5	**5**
5	**9**
5	**9**
9	**9**

AVERAGE MEAN DEVIATION (*AMD*)

Definition:

Formula for the average mean deviation (AMD): $\frac{\sum\left(X-\overline{X}\right)}{n}=\frac{\sum D}{n}$

Why does the average mean deviation always equals zero?

Disadvantage:

MEAN OF THE ABSOLUTE VALUE OF THE DEVIATIONS

- The absolute value of the deviations: $AMD = \frac{\sum \left| X - \overline{X} \right|}{N}$

Example: Absolute Value of the Deviations for Sample A

X	$(X - \bar{X})$	**D =** $X - \overline{X}$	$\lvert D \rvert = \lvert X - \overline{X} \rvert$
1			
5			
5			
5			
5			
5			
9			

$\sum D =$ $\sum \lvert D \rvert =$

$n =$ $n =$

AMD = $\lvert AMD \rvert =$

Disadvantage:

Alternative:

STANDARD DEVIATION

Symbols: σ (population) or *s* (sample).

Definitions:

1.

2.

Advantage:

Characteristics of standard deviation:

1.

2.

3.

4.

<u>Logic Behind The Standard Deviation Equations</u>

Goal:

Step 1:

- Determine the deviation, or distance from the mean, for each individual score
- Deviation score = X - μ (difference between a score & the mean)

e.g. Distribution of N = 5 scores: 1, 9, 5, 8, 7

$\sum X = 30$ $\qquad\qquad$ *Mean:* $\frac{30}{5} = 6 \rightarrow \mu =$

Score (X)	Deviation (X - μ)
1	
9	
5	
8	
7	

Step 2:
Square each deviation score

Score (X)	Deviation (X - μ)	Squared deviation $(X - \mu)^2$
1		
9		
5		
8		
7		
∑X=	∑(X - μ)=	$\sum(X - \mu)^2 =$

Step 3:
Use these squared values to compute the mean squared deviation, which is called variance.

e.g. For the set of N = 5 scores:

The sum of squared deviations =

The mean of the squared deviations (variance) is

VARIANCE

Definition:

Advantage:

Disadvantage:

Step 4:
Take the square root of the variance (a correction for having squared all the distances).

Standard deviation (s) = $\sqrt{\text{variance}} = \sqrt{s^2}$

The standard deviation is

FORMULAS FOR VARIANCE & STANDARD DEVIATION FOR POPULATIONS

Variance = mean squared deviation

$$= \frac{\text{sum of squared deviations}}{\text{number of scores}} = \frac{SS}{N}$$

Sum of squares (SS) = sum of the squared deviation scores

- Definitional formula: SS = $\sum (X - u)^2$

  Formula tells you to add up the squared deviations:
  1)
  2)
  3)
  4)

e.g. Compute SS for the set of N = 4 scores: 1, 0, 6, 1

X	**X - μ**	**$(X - \mu)^2$**	**Mean:**
1			$\mu = \sum X/N =$
0			
6			
1			
$\sum X =$	$\sum X =$	$\sum (X - \mu)^2 =$	

- *Definitional Formula*:
  - Advantages:

  - Disadvantages:

- *Computational Formula:*
    - Advantage:

$$SS = \sum X^2 - \frac{(\sum X)^2}{N}$$

X	X^2	SS
1		$SS = \sum X^2 - \frac{(\sum X)^2}{N}$
0		
6		
1		
$\sum X =$	$\sum X^2 =$	

Population Variance : $\sigma^2 = \frac{SS}{N} = \frac{\sum(X - \overline{X})^2}{N} \; or \; \frac{\sum X^2 - \frac{(\sum X)^2}{N}}{N}$

Population Standard Deviation: $\sigma = \sqrt{\frac{SS}{N}}$

Relationship Between Variance & Standard Deviation:

variance = (standard deviation)^2

standard deviation = $\sqrt{\text{var}iance}$

Sample Variance (s^2)

Definition:

Why samples consistently tend to be less variable than their populations:

Correction factor:

<u>Definitional Formula for Sample Variance (s^2):</u>

$$s^2 = \frac{\sum\left(X - \overline{X}\right)^2}{n-1} = \frac{SS}{n-1}$$

<u>Computational Formula for Sample Variance (s^2):</u>

$$s^2 = \frac{SS}{n-1} = \frac{\sum X^2 - \frac{\left(\sum X\right)^2}{n}}{n-1}$$

<u>Sample Standard Deviation (s):</u>

sample standard deviation = $s = \sqrt{\frac{SS}{n-1}} = \sqrt{variance}$

<u>Example:</u>

Sample of n = 7 scores from the population: 1, 6, 4, 3, 8, 7, 6

Score (X)	Deviation $\left(X - \overline{X}\right)$	Squared deviation $\left(X - \overline{X}\right)^2$	Computations
1			**M** =
6			
4			s^2=SS/(n-1)
3			

8			
7			$s = \sqrt{s^2}$
6			
$\sum X=$	$\sum D=$	SS=	

Practice Using Measures of Dispersion

Compute the variance & standard deviation for each sample

Sample A	Sample B
1	1
5	1
5	1
5	5
5	9
5	9
9	9

Mean A = Mean B =