Physics 202 Make-up Lab: Measuring Capacitance With the Bridge Technique

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Theoretical Discussion

Capacitors: Capacitors are passive electronic devices that store charge. Although there are many different capacitor designs, they are all conceptually equivalent to a simple parallel plate capacitor, depicted in figure 1A. The parallel plate capacitor consists of a pair of parallel conducting plates separated by an insulating material (such as air). When a potential difference is put across the leads of a capacitor, the plate that is connected to the more positive potential will acquire a charge $+Q$ and the plate that is connected to the more negative potential will acquire a charge $-Q$. The capacitance C of a capacitor is defined as the ratio between the absolute value of the charge acquired by either plate and the potential difference placed across it; i.e.,

$$C = \frac{|Q|}{V} \tag{1}$$

It is important to understand that the net charge on the capacitor is zero, and that the charge Q appearing in equation 1 is the absolute value of the charge on *either* plate of the capacitor. Furthermore, current does not flow between the plates of the capacitor. These plates are separated by an insulating gap. Charge separation occurs because electrons are drawn off of the plate connected to the positive end of the power supply and electrons are repelled by the negative end of the power supply onto the plate to which it is connected. Current does not flow across the gap[1].

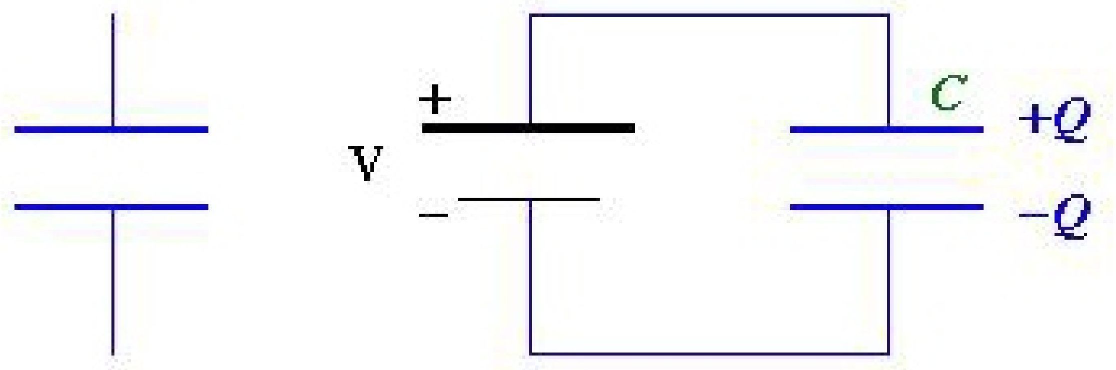

Figure 1A: A parallel plate capacitor

Figure 1B: A capacitor charged to a potential V_0

FIG. 1: Basic operation of a capacitor

Capacitors in series and parallel: When capacitors are connected in series, as shown in figure 2A, the same charge separations $\pm Q$ are across both capacitors. This must be so, otherwise charge would flow from one to the other until they were equal. Thus, the potential drop across a pair of capacitors C_1 and C_2 can be written as

$$\begin{aligned} V_1 &= \frac{Q}{C_1} \\ V_2 &= \frac{Q}{C_2} \end{aligned}$$

and the ratio of potential drops across them is equal to the reciprocal of the ratio of the capacitances; i.e.,

$$\frac{V_1}{V_2} = \frac{C_2}{C_1} \tag{2}$$

Recall that the corresponding equations for two resistors R_1 and R_2 in series are

$$\begin{aligned} V_1 &= IR_1 \\ V_2 &= IR_2 \end{aligned}$$

and since the same current flows through both resistors in a series circuit, the ratio of potential drops across two resistors in parallel is the ratio of the resistances,

$$\frac{V_1}{V_2} = \frac{R_1}{R_2} \tag{3}$$

By contrast, if two capacitors are connected in parallel, as in figure 2B, then the potential drop across both is the same; e.g.,

$$\begin{aligned} V_1 &= V_2 \\ &\Downarrow \\ \frac{Q_1}{C_1} &= \frac{Q_2}{C_2} \end{aligned}$$

so that the ratio of the charges on each capacitor is equal to the ratio of the capacitances; e.g.,

$$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} \tag{4}$$

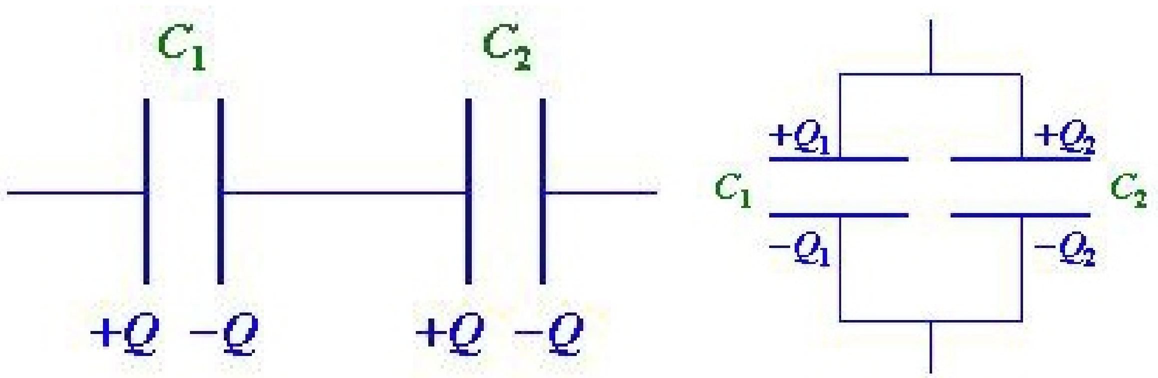

Figure 2A: Capacitors connected in series

Figure 2B: Capacitors connected in parallel

FIG. 2: Series and parallel configurations of capacitors

The bridge method of measuring capacitance: Consider the circuit depicted in figure 3[2]. In this circuit, a pair of resistors in series is connected to a pair of capacitors in series. The two (parallel) series circuits are themselves connected, between points A and B, by a line that runs through a pair of headphones. An AC (alternating-current) power supply is connected across the whole circuit.

Although the potential drop across the resistors must equal the potential drop across the capacitors at all times, in general the ratio of potential drops across resistors R_1 and R_2 will not be the same as the ratio of potential drops across capacitors C_{ref} and C_X. If these ratios are not the same, then potentials V_A and V_B will be different, and current can flow through the headphones, between points A and B. This will cause an audible tone in the headphones. By adjusting the ratio $\frac{R_1}{R_2}$, however, we can cause the ratio of potential drops across the resistors to equal the ratio of potential drops across the capacitors. In this case, potentials V_A and V_B will be the same, and current will stop flowing through the headphones, causing the audible tone to vanish. When this condition obtains, then according to equations 2 and 3,

$$\begin{aligned} \frac{V_{C_{ref}}}{V_{C_X}} &= \frac{V_{R_1}}{V_{R_2}} \\ &\Downarrow \\ \frac{C_X}{C_{ref}} &= \frac{R_1}{R_2} \\ &\Downarrow \\ C_X &= \frac{R_1}{R_2} C_{ref} \end{aligned}$$

and we can determine the unknown capacitance C_X from the known values of R_1, R_2, and C_{ref}.

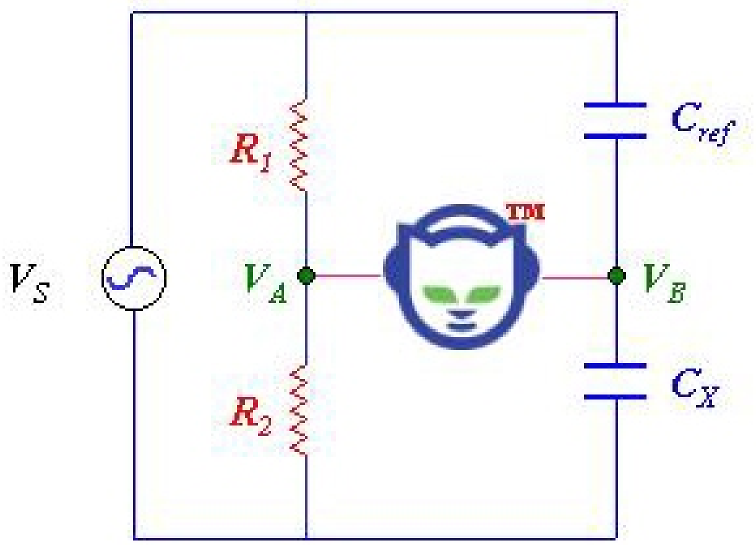

FIG. 3: A bridge circuit for measuring capacitance.

Experimental Procedure

The simplest means of varying the ratio between R_1 and R_2 is to use a device called a potentiometer as a voltage divider. A potentiometer consists of a single long resistor with a *wiper arm* that can be moved along the length of this resistor. If the total resistance of the potentiometer is R_{tot}, the resistance between the input lead and the wiper arm is R_1, and the resistance between the wiper arm and the output lead is R_2, where $R_1 + R_2 = R_{tot}$. A schematic diagram for conducting the experiment with a potentiometer is shown in figure 4.

The procedure for conducting the experiment is as follows:

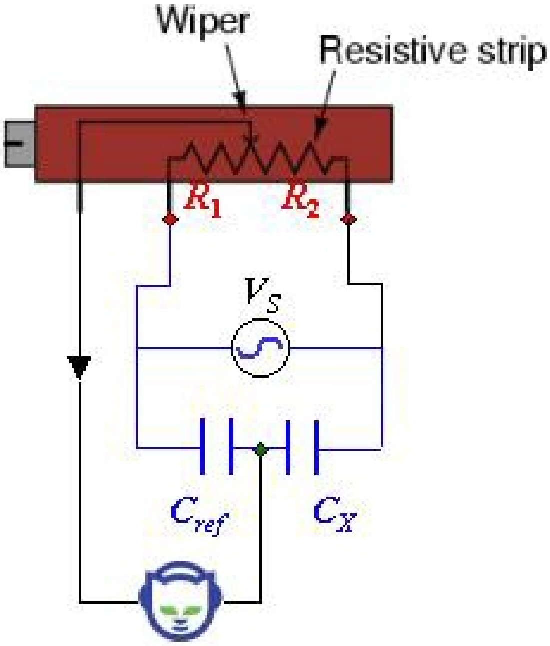

FIG. 4: A bridge circuit using a potentiometer.

1. Make sure the AC power supply, which is a function generator, is turned on. The frequency should be set somewhere in the range of a few kHz to obtain best results with the headphones[3]. You should hear a tone in your headphones if the bridge is unbalanced.

2. Move the wiper arm along the potentiometer until the tone goes away.

3. Disconnect the potentiometer from the circuit and, using the digital multimeter, measure R_1—the resistance between the input lead and the wiper arm, and R_2—the resistance between the wiper arm and the output lead.

4. Using the known value of C_{ref}, calculate the unknown capacitance C_X from the equation

$$C_X = \frac{R_1}{R_2} C_{ref}$$

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[1] If current does flow across the gap, you have broken your capacitor.

[2] The Napster logo shown in the figure is a registered trademark of Napster, Incorporated. Use of this logo in the bridge circuit is iconic and does not imply the use of Napster or any other file-sharing service in conducting this experiment. It is not the intent of the author to mock or disparage Napster in any way through the use of this symbol. It's just that I don't know how to draw headphones.

[3] In general, cheap headphones have much better response at high frequencies than at low. This is because they are inductively coupled devices whose impedance is proportional to the frequency. Healthy young people can hear frequencies over the range 20Hz–20kHz. The sensitivity of the human ear falls off as people age, particularly at higher frequencies. Feel free to set the frequency to your taste.