MAT 419/519 EXAM I
Spring 2003

Name:______________________________ SSN#:______________________________

1. Consider the equation $\frac{d}{dx}(x^2\frac{dy}{dx}) + (x^2 - 2)y = 0$. Apply the transformation $y(x) = \frac{u(x)}{\sqrt{x}}$ and thus find the solution.

2. Let $J_\mu = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!\Gamma(\mu + k + 1)}[\frac{x}{2}]^{\mu+2k}$. Show that $\frac{d}{dx}[\frac{J_\mu(x)}{x^\mu}] = -\frac{J_{\mu+1}(x)}{x^\mu}$, $(\mu \geq 0)$.

3. Let $f(x) \in L^2[a, b]$ and let $\{\phi_n\}_{n=1,2...}$ be a basis of $L^2[a, b]$. Show that the sequence of partial sums

$$f_k = \sum_{i=1}^{k} \langle f, \phi_i \rangle \phi_i$$

   is a Cauchy sequence and hence it converges in $L^2[a, b]$

4. Prove that the eigenvalues of a self adjoint operator are real and eigenvectors corresponding to different eigenvalues are orthogonal.

5. Show that $(d \circ d)(f) = 0$ for any real valued function f in $\mathbf{R}^3$.