Stoke's Theorem
Sample Problem:

Verify Stoke's theorem for the vector field
F = yz **i** + xz **j** + xy **k** over the surface S: $z = x^2 + y^2$ and $z \leq 1$.

Solution:
This is a very easy one.

Method 1.
The boundary of the surface is the curve C: $= x^2 + y^2 = 1$.
We first do the line integral. Begin all line integrals by computing the curl (as in the flow chart that I sent you). The result is 0. So the vector field is conservative and the work done around the close curve is 0. (In fact **F**=**grad**(xyz))

The Surface integral of **Curl F** · d**S** is trivially 0 since the Curl **F** = 0

So the two integrals give the same result.

Method 2.

In general the, if the vector field is not conservative, you would have to parametrize the curve and do the line integral
C: **r**(t) = cos t **i** + sin t **j** + 1 **k.**
d**r =** dx **i** + dy **j** + dz **k**
Evaluate **F** on the curve ie: compute **F**(**r**(t)).

$$\oint F \cdot dr = \int_0^{2\pi} (\sin t * 1) d(\cos t) + (\cos t * 1) d(\sin t) + (\sin t * \cos t) d(1)$$

$$\oint F \cdot dr = \int_0^{2\pi} (-\sin^2 t + \cos^2 t) dt = 0$$

In general, if the vector field is not conservative you would have to parametrize the surface and actually do the surface integral
In our case
r(u,v) = u **i** + v **j** + $(u^2 + v^2)$ **k.**
Then you would
1) Compute d**S** = ($\mathbf{r}_u$ x $\mathbf{r}_u$) du dv
2) Substitute x=u, y=v and $z = u^2 + v^2$ into Curl **F.**
3) Take the dot product of Curl **F** and ($\mathbf{r}_u$ x $\mathbf{r}_u$) du dv and integrate over the circle.

Of course the answer will still give you zero in this case since Curl **F**=0

NOTE!
1) If Div **G** = 0 then the flux ∫∫ (**G** · d**S**) is zero if the surface is closed, and it depends only on the boundary if it is not.
2) If **G** = curl **F** then div **G** = div (Curl **F**) =0. So, in the problem in question, you can forget about the paraboloid and replace it by the simplest surface with the same boundary, namely the plane z =1. This makes the integral very easy since

now d**S** is simply **k** dx dy, and the only component of the vector field that matters is the **k** component.