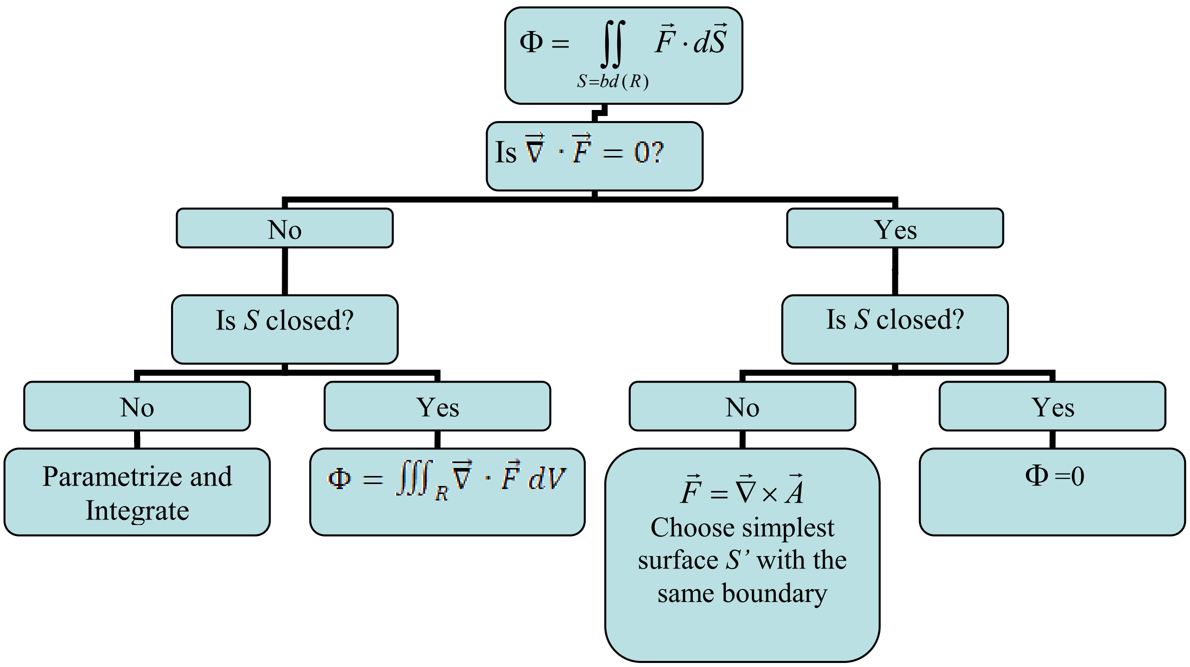
Hints on Surface Integrals



Case 1. Parametrize and Integrate
Write the parametric equation ***r****(u,v)* of the surface. Find $d\boldsymbol{S} = (\boldsymbol{r}_u \times \boldsymbol{r}_v)\, du\, dv$. Take the dot product with ***F*** and integrate over u and v.

Case 2. Use the Divergence (Gauss's) Theorem

a) The Divergence Theorem has exactly the same flavor as Stoke's Theorem.
   - You go up one dimension
   - You take some kind of derivative of the integrand
   - You integrate over the region bounded by the closed shape

b) In the xz plane, $d\vec{S} = (dx)\vec{i} \times (dz)\vec{k} = -(dxdz)\vec{j}$

c) In the yz plane, $d\vec{S} = (dy)\vec{j} \times (dz)\vec{k} = (dydz)\vec{i}$

Case 3.
In Stoke's theorem,

Curl (***F***) =0, implies that ***F*** = Grad *(f)* for some *f.*

Then it is easy to find the potential *f* and evaluate at the end-points. Another way of saying this is that the integral depends only on the boundary of the curve, namely the end-points. Thus, one could replace the curve by the simplest curve with the same boundary, namely a line joining the end-points. In practice, one never does this since it is simpler to evaluate *f* at the end-points.

In Gauss's theorem, if

Div(***F***)=0, implies that ***F***=Curl(***A***) for some ***A***.

Now it is not easy to find the vector potential **A**, but the situation is similar. The integral depends only on the boundary, so one can replace the original surface S with the simplest surface that has the same boundary. So, for example, if the surface is the upper hemisphere

$z = \sqrt{a^2 - x^2 - y^2}$

where the computation of the differential of surface is nasty, one could use the surface z=*0*, (where $d\boldsymbol{S} = dx\, dy\, \boldsymbol{k}$) and integrate over a circle of radius *a*.

Think of this as the butterfly net catcher theorem. If Div (***F***) =0 then there are no sources or sinks inside the surface, the flux of butterflies coming into the surface net depends only on the Rim of the net.

Case 4.

If Div(***F***)=0 and the surface is closed, then there are no sources or sinks inside the surface, so the net Flux is zero! Whatever goo comes in, must come out..

So Why is this Important?

Gauss's and Stoke's theorem are central to Physics. Without Gradients, Curls and Divergences, one could not give a full quantitative description of Gravitation, Electrodynamics, Fluid Dynamics, Aerospace Engineering, Atmospheric Sciences…you get the point.