Hints on Line Itegrals



Case 1. Parametrize and Integrate

- a) C is a line joining A and B: Use $\vec{r}(t) = \vec{A} + t(\vec{B} \vec{A})$
- b) C is a circle: Use $r(t) = (a \cos t)\vec{i} + (a \cos t)\vec{j} + (c)\vec{k}$
- c) C is an Ellipse: Use $r(t) = (a \cos t)\vec{i} + (b \cos t)\vec{j} + (c)\vec{k}$
- d) C is a Hyperbola: Use $r(t) = (a \cosh t)\vec{i} + (b \cosh t)\vec{j} + (c)\vec{k}$
- e) C is a Parabola: Use $r(t) = (t)\vec{i} + (at^2 + bt + c)\vec{j}$
- f) C is a Helix: Use $r(t) = (a \cos t)\vec{i} + (a \cos t)\vec{j} + (ct)\vec{k}$
- g) C is given by y=f(x). Use $r(t) = (t)\vec{i} + f(t)\vec{j}$

Case 2. Stoke's Theorem

- a) In R², Stoke's Theorem is called Green's Theorem. You might as well do every problem referring to Green's Theorem by using the general Stoke's theorem formula.
- b) In the xy plane , if F=<P,Q,0>, then ∇ X F=<0,0, Q_y-P_x>

c) In the xy plane, dS = <0,0,dx dy>, so
$$W = \iint_{R} \vec{\nabla} \times \vec{F} \cdot d\vec{S} = \iint_{R} (Q_{y} - P_{x}) dx dy$$

- d) In the xz plane, $d\vec{S} = (dx)\vec{i} \times (dz)\vec{k} = -(dxdz)\vec{j}$
- e) In the yz plane, $d\vec{S} = (dy)\vec{j} \times (dz)\vec{k} = (dydz)\vec{i}$

Case 3. Easy. Find the potential f and evaluate at the end points.