MATH 261 EXAM 3, Spring 2021

Simplify all answers. **Show your work!**

Name: Key

Score	
1	
2	
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1. a) Find $\int\int_R ye^{xy}\,dydx$; $R = [0, 2] \times [0, 2]$.

$I = \int_0^2 \int_0^2 y e^{xy}\,dx\,dy$

$= \int_0^2 e^{xy}\Big]_0^2 dy = \int_0^2 (e^{2y} - 1)\,dy$

$= \frac{1}{2}e^{2y} - y\Big]_0^2 = \frac{1}{2}e^4 - 2 - \frac{1}{2} = \boxed{\frac{1}{2}(e^4 - 5)}$

Ans:____________________.

b) Find $\int_0^2 \int_0^1 x/(y^2 + 1)\,dydx$

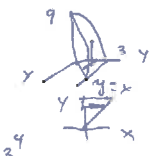
$I = \int_0^2 x \tan^{-1} y\Big]_0^1 dx = \frac{\pi}{4}\int_0^2 x\,dx$

$= \frac{\pi}{4} \cdot \frac{2^2}{2} = \boxed{\frac{\pi}{2}}$

Ans:____________________.

2. Find the volume in the first octant bounded by $z = 9 - y^2$ and $y = x$.

a) Set up the integral.



$I = \int_0^3 \int_0^y \int_0^{9-y^2} dz\,dx\,dy$

$= \int_0^3 \int_0^y (9 - y^2)\,dx\,dy$

$= \int_0^3 (9y - y^3)\,dy = 9 \cdot \frac{3^2}{2} - \frac{3^4}{4}$

$= 81\left(\frac{1}{2} - \frac{1}{4}\right) = \boxed{\frac{81}{4}}$

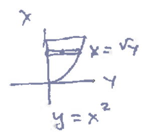
Ans:____________________.

b) Evaluate the integral.

Ans:____________________.

3. Let $I = \displaystyle\int_0^1 \int_{x^2}^1 x^3 e^{y^3}\,dydx$.

a) Reverse the order of integration.



$I = \int_0^1 \int_0^{\sqrt{y}} x^3 e^{y^3}\,dx\,dy$

$= \int_0^1 \frac{1}{4} y^2 e^{y^3}\,dy$

$= \frac{1}{4} \cdot \frac{1}{3} e^{y^3}\Big]_0^1 = \boxed{\frac{1}{12}(e - 1)}$

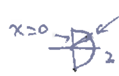
Ans:____________________.

b) Compute the integral.

Ans:____________________.

4. Let $I = \displaystyle\int\int_D xy\,dydx$, where D is the region bounded by $x = \sqrt{4 - y^2}$ and $x = 0$.

a) Convert to polar coordinates.



$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^2 (r\cos\theta)(r\sin\theta)\,r\,dr\,d\theta$

$= \frac{1}{2} \int_{-\pi/2}^{\pi/2} r^3 \sin 2\theta\,dr\,d\theta = \boxed{0}$

Ans:____________________.

b) Evaluate the integral.

Ans:____________________.

5. Convert the equation of the surface $z = \sqrt{3}r$ to:

a) Cartesian Coordinates.

Cone

$z = \sqrt{3}\sqrt{x^2 + y^2}$

Ans:____________________.

b) Spherical coordinates.

Find profile. Set $y = 0$ (Generator)

$z = \pm\sqrt{3}x$

Ans: $\theta = \pi/6$

Extra Space

Part II. | Name:

6. Let V be the volume bounded by the paraboloids $y = 3x^2 + 3z^2$ and $y = 4 - x^2 - z^2$.

a) Set up the volume integral.

$y = 3r^2 \qquad 3r^2 = 4 - r^2$

$y = 4 - r^2 \qquad r = 1$

$V = \int_0^{2\pi}\int_0^1\int_{3r^2}^{4-r^2} r\, dy\, dr\, d\theta$

$= \int_0^{2\pi}\int_0^1 r(4-4r^2)\, dr\, d\theta$

$= 2\pi \cdot 4\left(\frac{1}{2} - \frac{1}{4}\right) = \boxed{2\pi}$

Ans:__________________.

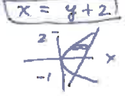
b) Compute the integral.

Ans:__________________.

7. A lamina with $\sigma = x^2 + 3y$ is bounded by $y = x - 2$ and $x = y^2$. Set up the integrals for:

a) The c.m. coordinate $\overline{x}$.

$\boxed{x = y + 2}$



set $y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = -1, 2$

$$x_{cm} = \frac{\iint x\, dm}{\iint dm} = \frac{\int_{-1}^{2}\int_{y^2}^{y+2} x(x^2+3y)\, dx\, dy}{\int_{-1}^{2}\int_{y^2}^{y+2} (x^2+3y)\, dx\, dy}$$

Ans:__________________.

b) The moment of inertia I_x.

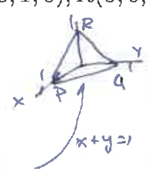
$I_x = \iint y^2\, dm$

$= \int_{-1}^{2}\int_{y^2}^{y+2} y^2(x^2+3y)\, dx\, dy$

Ans:__________________.

8. A tetrahedron has vertices at $P(1,0,0), Q(0,1,0), R(0,0,1)$ and $(0,0,0)$.

a) Write the equation of the plane PQR.



$\vec{V} = \overrightarrow{PQ} = \langle -1, 1, 0 \rangle$

$\vec{W} = \overrightarrow{PR} = \langle -1, 0, 1 \rangle$

$\vec{N} = \vec{V} \times \vec{W} = \langle 1, 1, 1 \rangle$

$1(x-1) + (y-1) + (z-1) = 0$

$\boxed{x + y + z = 1}$

Ans:__________________.

b) Set up an integral for the volume.

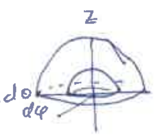
$V = \int_0^1\int_0^{1-x}\int_0^{1-x-y} dz\, dy\, dx = \boxed{\frac{1}{6}}$

$V = \frac{1}{6}\left|\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}\right|$

Ans:__________________.

9. Use spherical coordinates to evaluate $\int\int\int_E z\, dV$ where E is the region bounded by $z = \sqrt{1 - x^2 - y^2}$, $z = \sqrt{9 - x^2 - y^2}$, and $z = 0$.

a) Set up the integral.



$V = \int_0^{2\pi}\int_0^{\pi/2}\int_1^3 r\cos\theta\,(r^2 \sin\theta)\, dr\, d\theta\, d\varphi$

$= 2\pi\left(\frac{3^4}{4} - \frac{1^4}{4}\right) \cdot \frac{1}{2}\sin^2\theta\Big]_0^{\pi/2}$

$= 2\pi \cdot \frac{80}{4} \cdot \frac{1}{2} = \boxed{20\pi}$

Ans:__________________.

b) Compute the integral.

Ans:__________________.

10. Compute the Jacobian of the following transformations:

a) $x = uv$, $y = 4u + 3v$.

$dx = v\, du + u\, dv$

$dy = 4\, du + 3\, dv$

$dx \wedge dy = \left|\begin{matrix} v & u \\ 4 & 3 \end{matrix}\right| du \wedge dv$

Ans: $|J| = |3v - 4u|$

b) $x = uv$, $y = 4u + 3v$, $z = 5w$.

$|J| = \left|\begin{matrix} v & u & 0 \\ 4 & 3 & 0 \\ 0 & 0 & 5 \end{matrix}\right| = |5(3v - 4u)|$

Ans:__________________.

Extra space