Math 261 Exam 1, Spring 2021

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1. The points $P(2, 3, -4)$ and $Q(8, 3, 4)$ are endpoints of the diameter of a sphere. Find:

a) The radius of the sphere.

$r = \frac{1}{2} d(PQ) = \frac{1}{2}\sqrt{6^2+0^2+8^2} = 5$

Ans: $r = 5$

b) The equation of the sphere.

$C(5, 3, 0)$ Midpoint

Ans: $(x-5)^2 + (y-3)^2 + z^2 = 25$

2. Given the force $\mathbf{F}=\langle 2, 1, -3\rangle$ and the displacement $\mathbf{r}=\langle 4, -1, 1\rangle$, Find:

a) The component of $\mathbf{F}$ along $\mathbf{r}$.

$\|\vec{F}\| \cos\theta = \frac{\vec{F}\cdot\vec{r}}{\|\vec{r}\|} = \frac{8-1-3}{\sqrt{16+1+1}} = \frac{4}{\sqrt{18}} = \frac{4}{3\sqrt{2}} = \frac{2}{3}\sqrt{2}$

Ans:

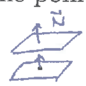
b) The work done by the force.

$W = \vec{F}\cdot\vec{r} = 4$

Ans: $W = 4$

3. Find the equation of the plane through the point $P(1, -2, 4)$ and:

a) parallel to $3x - 2y + z = 8$.

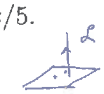


$\vec{N} = \langle 3, -2, 1\rangle$

$3(x-1) - 2(y+2) + 1(z-4) = 0$

Ans: $3x - 2y + z = 11$

b) Perpendicular to the line $x + 2 = 2y - 6 = z/5$.



$\frac{x+2}{1} = \frac{y-3}{1/2} = \frac{z}{5}$ $\quad \vec{v} = \langle 1, \frac{1}{2}, 5\rangle$

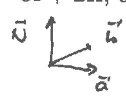
$\vec{N} = \langle 2, 1, 10\rangle$

$2(x-1) + (y+2) + 10(z-4) = 0$

Ans: $2x + y + 10z = 40$

4. Given the vectors $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + 2\mathbf{k}$, and $\mathbf{c} = 3\mathbf{j} - \mathbf{k}$, find:

a) A normal to $\mathbf{a}$ and $\mathbf{b}$.



$\vec{a} = \langle 2, -4, 2\rangle$

$\vec{b} = \langle 3, 0, 2\rangle$

$\vec{a}\times\vec{b} = \langle -8, 2, 12\rangle = -2\underbrace{\langle 4, -1, -6\rangle}_{N}$

$\vec{c} = \langle 0, 3, -1\rangle$

Ans: $N = \langle 4, -1, -6\rangle$

b) The volume of the parallepiped spanned by $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

$V = |(\vec{a}\,\vec{b}\,\vec{c})| = |(\vec{a}\times\vec{b})\cdot\vec{c}|$

$= |6 - 12|$

$= 6$

Ans: $V = 6$

5. Let $\mathcal{L}$ be the line of intersection of the planes $\mathcal{P}_1: \ 2x + y - z = 3$ and $\mathcal{P}_2: \ x - 2y + z = -1$. Find:

a) The point in $\mathcal{L}$ with $x = 0$.

$y - z = 3$

$-2y + z = -1$

$-y = 2$

$y = -2$

$z = -5$

Ans: $P(0, -2, -5)$

b) A parametric equation of $\mathcal{L}$.

$\vec{N}_1 = \langle 2, 1, -1\rangle$

$\vec{N}_2 = \langle 1, -2, 1\rangle$

$\vec{v} = \vec{N}_1\times\vec{N}_2 = \langle -1, -3, -5\rangle$

$x = 0 - t$

$y = -2 - 3t$

$z = -5 - 5t$

Ans:

Extra Space

Part II. | Name:

6. Identify the names of the graphs described by the following equations in $\mathbf{R}^3$.

a) $4x^2 = 4y^2 + z^2$ Ans: Elliptic Cone

b) $x^2 - 6y^2 - z^2 = 9$ Ans: Elliptic Hyperboloid of 2-sheets

c) $2x^2 + 4y^2 - 6z = 0$ Ans: Elliptic Paraboloid

d) $9y = z^2 - 4x^2$ Ans: Saddle

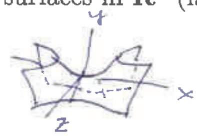
e) $\mathbf{r}(t) = \langle 7, 3\cosh t, 4\sinh t, \rangle$ Ans: Hyperbola

f) $\mathbf{r}(t) = \langle 8t, 4t^2 - 1, -2\rangle$ Ans: Parabola

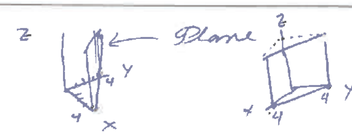
7. Describe and sketch the following surfaces in $\mathbf{R}^3$ (label the coordinate axes):

a) $y = 4x^2 - 9z^2$.

Saddle



b) $x + y = 4$



8. Let $\mathbf{r}(t) = \langle 2t, t^2, \frac{1}{3}t^3\rangle$, with $0 \le t \le 1$.

a) Find the length of the curve.

$\dot{\vec{r}} = \langle 2, 2t, t^2 \rangle$

$v = \sqrt{4 + 4t^2 + t^4}$

$= \sqrt{(2+t^2)^2}$

$= 2 + t^2$

$s = \int_0^1 (2 + t^2)\, dt$

$= 2 + \frac{1}{3} = \frac{7}{3}$

Ans: $s = 7/3$

9. Let $\mathbf{r}(t) = (2+t)\mathbf{i} + (4 - t^2)\mathbf{j} + (1/3)t^3\mathbf{k}$.

a) Find the speed at $t = 1$.

$\dot{\vec{r}} = \langle 1, -2t, t^2 \rangle$ $\qquad \dot{\vec{r}}(1) = \langle 1, -2, 1 \rangle$

$\ddot{\vec{r}} = \langle 0, -2, 2t \rangle$ $\qquad \ddot{\vec{r}}(1) = \langle 0, -2, 2 \rangle$

$v = \sqrt{6}$ at $t = 1$

Ans: $v = \sqrt{6}$ at $t = 1$

b) Find the unit tangent $\mathbf{T}$ at $t = 1$.

$\vec{T}(1) = \frac{1}{\sqrt{6}} \langle 1, -2, 1 \rangle$

Ans: $\vec{T}(1) = \frac{1}{\sqrt{6}} \langle 1, -2, 1 \rangle$

10. c) the curvature κ at $t = 1$.

$\dot{\vec{r}} \times \ddot{\vec{r}}\,\big|_{t=1} = \langle -2, -2, -2 \rangle = -2 \langle 1, 1, 1 \rangle$

$\|\dot{\vec{r}} \times \ddot{\vec{r}}\|_{t=1} = 2\sqrt{3}$

$\kappa(1) = \frac{2\sqrt{3}}{6\sqrt{6}} = \frac{1}{3\sqrt{2}} = \frac{\sqrt{2}}{6}$

Ans: $\kappa(1) = \frac{\sqrt{2}}{6}$

d) Find the centripetal acceleration a_N at $t = 1$.

$a_c = v^2 \kappa = 6 \cdot \frac{\sqrt{2}}{6} = \sqrt{2}$

Ans: $a_c(1) = \sqrt{2}$

Extra Space