

Does  $\sum_{n=1}^{\infty} a_n$  Converge ?

$a_n \rightarrow 0$

N → D

Y

$a_n = ar^n$

$|r| < 1$

Y → CA  
N → D

$a_n = b_n - b_{n-1}$

$b_n \rightarrow 0$

Y → C  
N → D

$a_n = f(n)$

$\int_1^{\infty} f(x) dx$

C → C  
D → D

Often used if terms contain  $\ln(n)$ .

$a_n < b_n$     $a_n > b_n$

Y →   Y →

$\sum_{n=1}^{\infty} b_n < \infty$     $\sum_{n=1}^{\infty} b_n = \infty$

Y → C   Y → D

Compare to Geometric and p-series

$\lim_{n \rightarrow \infty} \left| \frac{a_n}{b_n} \right| = c > 0$

Y

$\sum_{n=1}^{\infty} |b_n| < \infty$     $\sum_{n=1}^{\infty} |b_n| = \infty$

Y → CA   Y → D

Compare to Geom. and p-series. Test always works for ratios of polynomials or roots thereof.

$\rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$

$\rho < 1$     $\rho > 1$     $\rho = 1$

Y → CA   Y → D   Y → F

Use if there are exponentials and factorials. Test always fails if terms only contain ratios of polynomials or fractional exponents

$\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|} = \rho < 1$

$\rho < 1$     $\rho > 1$     $\rho = 1$

Y → CA   Y → D   Y → F

Use if there are powers of n but no factorials.

If CA fails and series alternates, test CC

1. Does  $a_n = (-1)^n b_n$  ?
2. Does  $b_n \rightarrow 0$  ?
3. Is  $b_n$  decreasing ?