

Show all work!	Name: <u>Key</u>	Score																															
<p>1. The displacement of an object thrown upwards in Mars is given by $y = 10t - 1.86t^2$. Find the average velocity:</p> <p>a) In the interval $[2, 3]$.</p> <table style="margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 2px;">t</td> <td style="padding: 2px;">y</td> <td rowspan="3" style="padding-left: 20px;">$v_{AV} = \frac{13.26 - 12.56}{3 - 2}$</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">2</td> <td style="padding: 2px;">12.56</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">3</td> <td style="padding: 2px;">13.26</td> </tr> <tr> <td colspan="3" style="text-align: center; padding-top: 10px;">$= 0.7$</td> </tr> </table> <p style="text-align: right; margin-right: 20px;">Ans: _____</p>	t	y	$v_{AV} = \frac{13.26 - 12.56}{3 - 2}$	2	12.56	3	13.26	$= 0.7$			<p>b) In the interval $[2, 2.1]$.</p> <table style="margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 2px;">t</td> <td style="padding: 2px;">y</td> <td rowspan="3" style="padding-left: 20px;">$v_{AV} = \frac{12.797 - 12.56}{2.1 - 2}$</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">2</td> <td style="padding: 2px;">12.56</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">2.1</td> <td style="padding: 2px;">12.797</td> </tr> <tr> <td colspan="3" style="text-align: center; padding-top: 10px;">$= 2.377$</td> </tr> </table> <p style="text-align: right; margin-right: 20px;">Ans: _____</p>	t	y	$v_{AV} = \frac{12.797 - 12.56}{2.1 - 2}$	2	12.56	2.1	12.797	$= 2.377$			<table border="1" style="margin: auto; border-collapse: collapse;"> <tr><td style="width: 20px; height: 20px;">1</td></tr> <tr><td style="width: 20px; height: 20px;">2</td></tr> <tr><td style="width: 20px; height: 20px;">3</td></tr> <tr><td style="width: 20px; height: 20px;">4</td></tr> <tr><td style="width: 20px; height: 20px;">5</td></tr> <tr><td style="width: 20px; height: 20px;">6</td></tr> <tr><td style="width: 20px; height: 20px;">7</td></tr> <tr><td style="width: 20px; height: 20px;">8</td></tr> <tr><td style="width: 20px; height: 20px;">9</td></tr> <tr><td style="width: 20px; height: 20px;">10</td></tr> <tr><td style="width: 20px; height: 20px;">Tot</td></tr> </table>	1	2	3	4	5	6	7	8	9	10	Tot
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<p>2. Guess the limit. Show a table or a graph as appropriate.</p> <p>a) $\lim_{x \rightarrow 0} (e^x - 1)/x$. (Numerically)</p> <table style="margin-left: 20px;"> <tr> <td style="border-right: 1px solid black; padding: 2px;">x</td> <td style="padding: 2px;">$f(x)$</td> <td rowspan="7" style="padding-left: 20px; vertical-align: middle;">$\rightarrow 1$</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">0.1</td> <td style="padding: 2px;">1.0517</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">0.01</td> <td style="padding: 2px;">1.005</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">0.001</td> <td style="padding: 2px;">1.0005</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">-0.001</td> <td style="padding: 2px;">0.9995</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">-0.01</td> <td style="padding: 2px;">0.995</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 2px;">-0.1</td> <td style="padding: 2px;">0.956</td> </tr> </table> <p style="text-align: right; margin-right: 20px;">Ans: <u>1</u></p>	x	$f(x)$	$\rightarrow 1$	0.1	1.0517	0.01	1.005	0.001	1.0005	-0.001	0.9995	-0.01	0.995	-0.1	0.956	<p>b) $\lim_{x \rightarrow 2^+} [x/(x-2)]$. (Graphically)</p> <p style="margin-left: 20px;">$= \infty$</p> <p style="text-align: right; margin-right: 20px;">Ans: _____</p>																	
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<p>3. Evaluate the limit analytically, if it exists:</p> <p>a) $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x - 3} = \lim_{x \rightarrow 3} \frac{(x-3)(x+5)}{(x-3)}$</p> <p style="margin-left: 40px;">$= \lim_{x \rightarrow 3} (x+5)$</p> <p style="margin-left: 40px;">$= 8$</p> <p style="text-align: right; margin-right: 20px;">Ans: _____</p>	<p>b) $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9} = \lim_{x \rightarrow 9} \left(\frac{\sqrt{x} - 3}{x - 9} \right) \left(\frac{\sqrt{x} + 3}{\sqrt{x} + 3} \right)$</p> <p style="margin-left: 40px;">$= \lim_{x \rightarrow 9} \frac{x - 9}{(x-9)(\sqrt{x} + 3)} = \lim_{x \rightarrow 9} \frac{1}{\sqrt{x} + 3}$</p> <p style="margin-left: 40px;">$= \frac{1}{6}$</p> <p style="text-align: right; margin-right: 20px;">Ans: _____</p>																																
<p>4. Find the following limits:</p> <p>a) $\lim_{x \rightarrow \infty} \frac{2x^2 + 1}{3x^2 - 5} = \frac{2}{3}$</p> <p style="text-align: right; margin-right: 20px;">Ans: _____</p>	<p>b) $\lim_{x \rightarrow \infty} \frac{\sqrt{4x^6 + 9}}{3x^3 + 7} = \frac{2}{3}$</p> <p style="text-align: right; margin-right: 20px;">Ans: _____</p>																																
<p>5. Using the ϵ, δ definition, prove rigorously that: $\lim_{x \rightarrow 2} (5x - 7) = 3$.</p> <p>Let $\delta = \frac{\epsilon}{5}$. Assume $x - 2 < \delta$</p> <p>That is, $x - 2 < \frac{\epsilon}{5}$.</p> <p>Then $f(x) - L = 5x - 7 - 3$</p> <p style="margin-left: 40px;">$= 5x - 10$</p> <p style="margin-left: 40px;">$= 5 x - 2$</p> <p style="margin-left: 40px;">$< 5 \cdot \frac{\epsilon}{5}$</p> <p style="margin-left: 40px;">$< \epsilon$.</p>																																	
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6. Explain why each of the following functions are discontinuous at $x = 1$.

a) $f(x) = \frac{1}{x-1}$.

$f(1)$ not defined

b) $f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1} & x \neq 1 \\ 1 & x = 1 \end{cases}$

a) $f(1) = 1$
 $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{x(x-1)}{(x-1)(x+1)}$

$= \lim_{x \rightarrow 1} \frac{x}{x+1} = \frac{1}{2}$

$\lim_{x \rightarrow 1} f(x) \neq f(1)$

7. Find the derivative of the function using the limit definition of derivative:

$f(x) = \frac{1}{x+4}$ $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

$f'(x) = \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{1}{x+h+4} - \frac{1}{x+4} \right]$
 $= \lim_{h \rightarrow 0} \frac{1}{h} \left[\frac{(x+4) - (x+h+4)}{(x+h+4)(x+4)} \right]$
 $= \lim_{h \rightarrow 0} \frac{1}{h} \frac{-h}{(x+h+4)(x+4)} = \frac{-1}{(x+4)^2}$

Ans: _____.

8. Differentiate the following functions:

a) $f(x) = \sqrt{7x} - 3e^{x^2}$
 $= \sqrt{7} x^{1/2} - 3e^{x^2}$
 $f'(x) = \sqrt{7} \cdot \frac{1}{2} x^{-1/2} - 3 \cdot e^{x^2} (2x)$
 $= \frac{\sqrt{7}}{2\sqrt{x}} - 6x e^{x^2}$

Ans: _____.

b) $f(x) = \sqrt{x^2 - 9} = (x^2 - 9)^{1/2}$
 $f'(x) = \frac{1}{2} (x^2 - 9)^{-1/2} \cdot (2x)$
 $= \frac{x}{\sqrt{x^2 - 9}}$

Ans: _____.

9. Differentiate the following functions:

a) $f(x) = x^2 e^{3x} = x^2 \cdot e^{3x} (3) - 2x e^{3x}$
 $= x(3x - 2) e^{3x}$

Ans: _____.

b) $f(x) = \frac{x^2 - 4}{x^2 + 4}$
 $f'(x) = \frac{(x^2 + 4)2x - (x^2 - 4)2x}{(x^2 + 4)^2}$
 $= \frac{2x [(x^2 + 4) - (x^2 - 4)]}{(x^2 + 4)^2}$

Ans: $f'(x) = \frac{16x}{(x^2 + 4)^2}$

10. Differentiate the following functions:

a) $f(x) = \sin^4(2x)$
 $f'(x) = 4 \sin^3(2x) \cdot \frac{d}{dx} \sin(2x)$
 $= 4 \sin^3(2x) \cdot \cos(2x) \cdot 2$
 $= 8 \cos 2x \sin^3 2x$

Ans: _____.

b) $f(x) = \sec(2e^x)$
 $f'(x) = \sec(2e^x) \tan(2e^x) \cdot \frac{d}{dx} (2e^x)$
 $= 2e^x \sec(2e^x) \tan(2e^x)$

Ans: _____.

Extra space.