Math 418 Review Test II

Be able to define the Fourier transform and inverse Fourier Transform for functions

$$\hat{f}(w) = \mathcal{F}(f(x))(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx \text{ and } \mathcal{F}^{-1}(\hat{f}(w))(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(w) e^{iwx} dw$$

Be able to compute a fourier transform (sinc, gaussian,$e^{-|x|}$). Know the **definition of convolution** $f * g(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x-t) g(t)\, dt$ for functions on defined the real line. Know these basic properties and be able to use them. **Be able to derive 1-7** (Section 7.2)

1) $\mathcal{F}(f(x) + g(x))(w) = \hat{f}(w) + \hat{g}(w)$ 2) $\mathcal{F}(cf(x))(w) = c\hat{f}(w)$

3) $\mathcal{F}(e^{ikx} f(x))(w) = \hat{f}(w-k)$ 4) $\mathcal{F}(f(x-k))(w) = e^{-ikw} \hat{f}(w)$

5) $\mathcal{F}(f'(x))(w) = iw\hat{f}(w)$ 6) $\mathcal{F}(xf(x))(w) = i\frac{d}{dw}\hat{f}(w)$

7) $\mathcal{F}(f * g)(w) = \hat{f}(w) \cdot \hat{g}(w)$ 8) $\mathcal{F}(\mathcal{F}(f))(x) = f(-x)$

Examples: Compute $\mathcal{F}(1_{[-1/2,1/2)}(x))(w)$ using the definition. Show $\mathcal{F}(f(ax))(w) = \frac{1}{a}\hat{f}(\frac{w}{a})$ for $a > 0$ and use these two to find $\mathcal{F}(1_{[-1,1)}(x))(w)$.

Be able to convert a Partial Differential equation into an ordinary one and solve.

Example: Convert and solve the PDE $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $u(x,0) = f(x)$ by transforming it into an ODE (Also examples 1-5 sec 7.3) .

Be able to derive(and apply) the the solution to the heat equation using Gauss's kernel $g_t(x) = \frac{1}{\sqrt{2t}} e^{-x^2/4t}$ (7.4) pg 421

Be able to derive(and apply) the the solution to the Dirchlet Problem on the upper half-plane using the Poisson kernel $P_y(x) = \sqrt{\frac{2}{\pi}} \frac{y}{x^2 + y^2}$ (7.5) pg 430

Be able to take/**DEFINE** derivatives and Fourier Transforms of generalized functions and be able to apply the operational properties. Example 6 pg453

Know the definition of the Laplace transform, exponential order and convolution in this setting. Know how to derive, Laplace transforms of $1, t, \cos(t), \sin(t), e^{\alpha t}$ and be able to prove the properties 1-5 below.

$1)\mathcal{L}(f(t)+g(t))(w) = F(s)+G(s)$	$2)\mathcal{L}(cf(t))(w) = cF(s)$
$3)\mathcal{L}(e^{\alpha t}f(t))(s) = F(s-\alpha)$	$4)\mathcal{L}(\mathcal{U}_0(t-a)f(t-a)))(s) = e^{-as}F(s)$
$5)\mathcal{L}(f'(t))(s) = s\mathcal{L}(f(s)) - f(0)$	$6)\mathcal{F}(tf(t))(w) = \frac{d}{ds}F(s)$
$7)\mathcal{L}(f * g)(s) = F(s)\cdot G(s)$	$8)\mathcal{L}(\delta_0(t-a)) = e^{-as}$

Be able to solve one variable differential equations. Using the Laplace transform. Concentrate on Applying the properties of the Transform and not finding specific solutions via a tables, maple etc. Example 8 page 489