Math 418/518 Homework 3

Directions: NEATLY write all solutions on your own paper. Use may use Maple or another computer program to find the solutions. Include any print-outs if you use a CAS.

1) Another representation of the the Bessel function $J_n(x) = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} e^{i(x\sin(\theta)-n\theta)}d\theta.$ Use this to show that

$$1 = |J_0(x)|^2 + 2\sum_{n=1}^{\infty} |J_n(x)|^2$$

and conclude $\lim_{n\to\infty} J_n(x) = 0$. Hint use Parseval's Identity.

2) Use the Fourier transform to find the solution to the differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}, \quad u(x,0) = f(x).$$

$$-\infty < x < \infty \qquad\qquad t > 0$$

Hint: Use properties of the Fourier transform.

3) Prove the Semigroup property of the Poisson Kernel. That is, use the Fourier transform to show $P_{y_1} * P_{y_2}(x) = P_{y_1+y_2}(x)$

4) Show that $\displaystyle\int_{\mathbb{R}} f(x)\overline{g(x)}dx = \int_{\mathbb{R}} \hat{f}(w)\overline{\hat{g}(w)}dw$ and conclude that the Fourier transform is norm preserving, i.e. $\displaystyle\int_{\mathbb{R}} |f(x)|^2 dx = \int_{\mathbb{R}} |\hat{f}(w)|^2 dw$.

5)Use properties of the Fourier transform to verify that D'Alembert's Solution is valid for the wave equation on an infinite string. See prob 21 page 7.3