Math 418-518 Homework 2

Directions: NEATLY write all solutions on your own paper. You may use Maple or another computer program to find the solutions to 3 and 5. Include some printouts of the graphs requested or email an animation as a file.

1)Produce the solution of the two dimensional heat equation on page 161 of your text.

2) Show that

$$u(x,y) = \frac{1}{2\pi}\int_0^{2\pi} f(\phi)\frac{1-x^2-y^2}{1+2x\cos\phi+2y\sin\phi+x^2+y^2}d\phi$$

and

$$u(x,y) = \frac{1}{2\pi}\int_0^{2\pi} f(\phi)\frac{1-|z|^2}{|e^{i\phi}-z|^2}d\phi$$

are alternate forms of the Poisson integral formula on the unit disk in cartesian coordinates. See page 227 and recall $z = re^{i\theta} = x + iy$

3) Solve the $\nabla^2 u = f(x,y)$ on the rectangle $[0,1]\times[0,1]$ with $f(x,y) = xy$ and $u(0,y) = u(x,0) = u(a,y) = u(x,b) = 0$.

4) We say that a function $u(x,y)$ is harmonic if, $\nabla^2 u = 0$. Show that if u and u^2 are both harmonic, then u is a constant

5) Solve the two dimensional Wave equation. Give the value of the coefficients $B_{1,1}, B_{1,2}, B_{2,2}, B_{2,1}, B_{2,2}, B^*_{1,1}, B^*_{1,2}, B^*_{2,1}, B^*_{2,2}$.

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, 0 < x < 1,\ 0 < y < 1,\ t > 0,$$

$$u(0,y,t) = u(1,y,t) = u(x,0,t) = u(x,1,t) = 0,$$

$$u(x,y,0) = x(1-x)y(1-y),\ \frac{\partial u}{\partial t}(x,y,0) = x(1-y).$$

Plot a partial sum of the series solution at times $t = 0,1,2,3$ to show the vibrations of the membrane, or plot a 3d animation and send it to me.