Math 418/518 Homework 2

Directions: NEATLY write all solutions on your own paper. Solutions should include details like integration by parts and reasons for convergence or divergence. 418, 1-4 518, 1-5

1) Another representation of the the Bessel function $J_n(x) = \dfrac{1}{2\pi}\displaystyle\int_0^{2\pi} e^{i(x\sin(\theta)-n\theta)}d\theta$. Use this to show that

$$1 = |J_0(x)|^2 + 2\sum_{n=1}^{\infty}|J_n(x)|^2$$

and conclude $\lim\limits_{n\to\infty} J_n(x) = 0$. Hint use Parseval's Identity.

2) Show that $\displaystyle\int_{\mathbb{R}} f(x)\overline{g(x)}dx = \int_{\mathbb{R}} \hat{f}(w)\overline{\hat{g}(w)}dw$ and conclude that the Fourier transform is norm preserving, i.e. $\displaystyle\int_{\mathbb{R}} |f(x)|^2dx = \int_{\mathbb{R}} |\hat{f}(w)|^2dw$.

3) We say that a function $u(x, y)$ is harmonic if, $\nabla^2 u = 0$. Show that if u and u^2 are both harmonic, then u is a constant

4) Use the Fourier transform to find the solution to the differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x}, \ u(x,0) = f(x).$$

$$-\infty < x < \infty \qquad\qquad t > 0$$

Hint: Use properties of the Fourier transform.

5)Show that

$$u(x,y) = \frac{1}{2\pi}\int_0^{2\pi} f(\phi)\frac{1-x^2-y^2}{1-2x\cos\phi - 2y\sin\phi + x^2 + y^2}d\phi$$

and

$$u(x,y) = \frac{1}{2\pi}\int_0^{2\pi} f(\phi)\frac{1-|z|^2}{|e^{i\phi}-z|^2}d\phi$$

are alternate forms of the Poisson integral formula on the unit disk in cartesian coordinates. See page 227 and recall $z = re^{i\theta} = x + iy$