Math 418/518 Homework 1

Directions: NEATLY write all solutions on your own paper. Solutions should include details like integration by parts and reasons for convergence or divergence. All expansions are being done on the interval $[-\pi, \pi)$

1) Find the complex Fourier series expansion ($N = -4..4$) for $f(x) = x$, $0 < x < 2\pi$. Does the sum of the coefficients, c_n converge in magnitude?

2) The Fourier series expansion for $f(x) = x^2$ on $[-\pi, \pi]$ is $x^2 = \dfrac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \dfrac{(-1)^n}{n^2} \cos(nx)$.

Use Parseval's identity to find $\sum_{n=1}^{\infty} \dfrac{1}{n^4}$.

3) Define the **saw tooth function** $f(x)$ to be

$$f(x) = \begin{cases} \frac{1}{2}(-\pi - x) & -\pi \leq x \leq 0 \\ \frac{1}{2}(\pi - x) & 0 < x \leq \pi \end{cases} .$$

Your book (pg 29) shows that the partial sums of the Fourier series are $s_N(x) = \sum_{n=1}^{N} \dfrac{\sin(nx)}{n}$.

Show that the term by term derivative of the Fourier series does not converge to $f'(x)$ for all x.

4) Let $D_N(x) = \sum_{n=-N}^{N} e^{inx}$ be the complex form of the Dirichlet Kernel.

a) Show that $\dfrac{1}{2\pi}\int_{-\pi}^{\pi} D_N(x)e^{-imx}dx = 1$ if $|m| \leq N$ and 0 otherwise. Conclude that $\dfrac{1}{2\pi}\int_{-\pi}^{\pi} |D_N(x)|^2 dx = 2N + 1$.

b) Use the Dirichlet Kernel and the identity $S_N(x) = \dfrac{1}{2\pi}\int_0^{2\pi} f(x-t)D_N(t)dt$ to recompute the N^{th} partial Fourier series for the $f(x) = x$ in problem # 1. Reconcile that they are the same answer.

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5) Use a geometric series argument to show $D_N(x) = \dfrac{e^{(N+1)ix} - e^{-iNx}}{e^{ix} - 1}$ multiply both top and bottom by $e^{-ix/2}$ and conclude that $D_N(X) = \dfrac{\sin(N + \frac{1}{2})x}{\sin(\frac{x}{2})}$.