Basics and Properties

Given linear ODE $a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots a_1 y' + a_0 y = 0$
with λ_i solutions of characteristic polynomial, general solution for distinct roots

$$y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} + \cdots + c_n e^{\lambda_n x}$$

Bessel $xy'' + y' + \lambda^2 xy = 0$, $y = d_1 J_0(\alpha x) + d_2 Y_0(\alpha x)$. J_0 and Y_0 are the **Bessel functions of order 0 of the first and second kind.** We denote the zeros of J_0 by α_n

Euler's Equation $x^2 y'' + xy' - n^2 y = 0$

$$y = c_1 x^n + c_2 x^{-n}, \text{ for } n = 1, 2, 3 \ldots \text{ and } y = c_1 + c_2 \ln(r), \text{ for } n = 0$$

$$1)\mathcal{F}(f(x) + g(x))(w) = \hat{f}(w) + \hat{g}(w) \qquad 2)\mathcal{F}(cf(x))(w) = c\hat{f}(w)$$

$$3)\mathcal{F}(e^{ikx} f(x))(w) = \hat{f}(w - k) \qquad 4)\mathcal{F}(f(x - k))(w) = e^{-ikw}\hat{f}(w)$$

$$5)\mathcal{F}(f'(x))(w) = iw\hat{f}(w) \qquad 6)\mathcal{F}(xf(x))(w) = i\frac{d}{dw}\hat{f}(w)$$

$$7)\mathcal{F}(f * g)(w) = \hat{f}(w) \cdot \hat{g}(w) \qquad 8)\mathcal{F}(\mathcal{F}(f))(x) = f(-x)$$

$$9)\mathcal{F}(\delta_\alpha)(w) = \frac{e^{-i\alpha w}}{\sqrt{2\pi}} \qquad 10)\mathcal{F}(\mathcal{U}_\alpha)(w) = \frac{-ie^{-i\alpha w}}{\sqrt{2\pi} w}$$

$$1)\mathcal{L}(f(t) + g(t))(w) = F(s) + F(s) \qquad 2)\mathcal{L}(cf(t))(w) = cF(s)$$

$$3)\mathcal{L}(e^{\alpha t} f(t))(s) = F(s - \alpha) \qquad 4)\mathcal{L}(\mathcal{U}_0(t - a) f(t - a)))(s) = e^{-as} F(s)$$

$$5)\mathcal{L}(f'(t))(s) = s\mathcal{L}(f(s)) - f(0) \qquad 6)\mathcal{F}(tf(t))(w) = -\frac{d}{ds}F(s)$$

$$7)\mathcal{F}(f * g)(s) = F(s) \cdot G(s) \qquad 8)\mathcal{L}(\delta_0(t - a)) = e^{-as}$$

Sturm-Louiville $[p(x)y']' + [q(x) + \lambda r(x)]y = 0$

Green's Identities

$$\iint_\Omega \left(u\nabla^2 v + \nabla u \cdot \nabla v\right) dA = \int_\Gamma u\frac{\partial v}{\partial n} ds, \quad \iint_\Omega \left(u\nabla^2 v - v\nabla^2 u\right) dA = \int_\Gamma \left(u\frac{\partial v}{\partial n} - v\frac{\partial u}{\partial n}\right) ds$$

Normal derivative $\dfrac{\partial u}{\partial n} = \nabla u \cdot \overrightarrow{n} = \langle u_x, u_y \rangle \cdot \dfrac{\langle -y'(t), x'(t) \rangle}{\sqrt{y'(t)^2 + x'(t)^2}}$

Cauchy Integral Formula : $f(a) = \dfrac{1}{2\pi i} \oint_C \dfrac{f(z)}{z - a} dz$

Poisson Integral unit disk & upper half-plane

$$u(r, \theta) = \frac{1 - r^2}{2\pi} \int_{2\pi}^{0} \frac{f(\theta)}{(1 + r^2 - 2 * r\cos(\theta - \phi))} d\phi \qquad u(x, y) = \frac{y}{\pi} \int_{-\infty}^{\infty} \frac{f(t)}{(t - x)^2 + y^2} dt$$

Green's Function unit disk & upper half-plane

$$G(z, z_0) = \ln\left|\frac{z - z_0}{1 - \overline{z_0} z}\right| \qquad G(z, z_0) = \ln\left|\frac{z - z_0}{z - \overline{z_0}}\right|$$