519 Review Test I

Chapter 6 Generalized Fourier Series(Thm 1 and Thm 2, 6.1), orthogonality with respect to weight, Parseval's Identity, Sturm-Liouville $[p(x)y']' + [q(x) + \lambda r(x)]y = 0$, Hermite example page 598

Proof of Theorrem 2 pg 336

Sample Problems 6.1: Exercise-6,7 6.2: Exercise-17,Example-4

Chapter 4 Wave equation in polar coordinates. Laplace equation in polar and cylindrical coordinates. Be able to use separation of variable to reduce the PDEs into ODEs. Use Sturm-Liouville to find orthogonal eigen functions of solutions and produce series solutions. 4.2 Theorem 1 4.3 Theorem 1, page 213 4.3 Theorem 1, page 213 4.4 Theorem 1, page 218 4.5 Theorem 1, page 228

Sample Problems 4.2: Example-1 ,4.3: Example-1 ,4.4: Exercise-29 a,Example-1 , 4.5: Exercises-1

Chapter 8 Laplace transform. $F(s) = \mathcal{L}(f(t))(s) = \int_0^\infty f(t)e^{-st}dx$. Be able to derive operational Properties.(see page two of review). Be able to use definition to derive basic Transforms. Turn PDE to ODE and solve. Convolution on $[0, \infty)$.

Sample Problems 8.1:Example-2,3,8 8.2: Exercise-39 Example-6,8 8.3: Exercise-14,15 Example-1,3

Basics and Properties

Given linear ODE $a_n y^{(n)} + a_{n-1}y^{(n-1)} + \cdots a_1 y' + a_0 y = 0$
with λ_i solutions of characteristic polynomial, general solution is
.

$$y = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} + \cdots + c_n e^{\lambda_n x}$$

Bernoulli zero $xy'' + y' + \lambda^2 xy = 0$, $y = d_1 J_0(\alpha x) + d_2 Y_0(\alpha x)$. J_0 and Y_0 are the **Bessel functions of order 0 of the first and second kind.** We denote the zeros of J_0 by α_n

Euler's Equation $x^2 y'' + xy' - n^2 y = 0$

$$y = c_1 x^n + c_2 x^{-n}, \text{ for } n = 1, 2, 3 \ldots$$

$$y = c_1 + c_2 \ln(r), \text{ for } n = 0$$

$$1)\mathcal{F}(f(x) + g(x))(w) = \hat{f}(w) + \hat{g}(w) \qquad 2)\mathcal{F}(cf(x))(w) = c\hat{f}(w)$$

$$3)\mathcal{F}(e^{ikx}f(x))(w) = \hat{f}(w-k) \qquad 4)\mathcal{F}(f(x-k))(w) = e^{-ikw}\hat{f}(w)$$

$$5)\mathcal{F}(f'(x))(w) = iw\hat{f}(w) \qquad 6)\mathcal{F}(xf(x))(w) = i\frac{d}{dw}\hat{f}(w)$$

$$7)\mathcal{F}(f * g)(w) = \hat{f}(w) \cdot \hat{g}(w) \qquad 8)\mathcal{F}(\mathcal{F}(f))(x) = f(-x)$$

$$9)\mathcal{F}(\delta_\alpha)(w) = \frac{e^{-i\alpha w}}{\sqrt{2\pi}} \qquad 10)\mathcal{F}(\mathcal{U}_\alpha)(w) = \frac{-ie^{-i\alpha w}}{\sqrt{2\pi}w}$$

$$1)\mathcal{L}(f(t) + g(t))(w) = F(s) + F(s) \qquad 2)\mathcal{L}(cf(t))(w) = cF(s)$$

$$3)\mathcal{L}(e^{\alpha t}f(t))(s) = F(s-\alpha) \qquad 4)\mathcal{L}(\mathcal{U}_0(t-a)f(t-a)))(s) = e^{-as}F(s)$$

$$5)\mathcal{L}(f'(t))(s) = s\mathcal{L}(f(s)) - f(0) \qquad 6)\mathcal{F}(tf(t))(w) = -\frac{d}{ds}F(s)$$

$$7)\mathcal{F}(f * g)(s) = F(s) \cdot G(s) \qquad 8)\mathcal{L}(\delta_0(t-a)) = e^{-as}$$

$$\text{Sturm-Louiville } [p(x)y']' + [q(x) + \lambda r(x)]y = 0$$