Math 519 Homework 1

Directions: NEATLY write all solutions on your own paper. Solutions should include details like integration by parts and reasons for convergence or divergence.

1) Show that $L_4(x) = \frac{1}{24}(x^4 - 16x^3 + 72x^2 - 96x + 24)$ and $L_1(x) = -x + 1$ are solutions of Laguerre's differential equation, $xy'' + (1-x)y' + ny = 0,\ 0 < x < \infty$, for $n = 4$ and $n = 1$ respectively. Use the weight $r(x) = e^{-x}$ to turn this into a Sturm -Liouville problem and conclude $\int_0^\infty L_4(x)L_1(x)e^{-x}dx = 0$.

2) Use separation of variables to find product solutions (example pg 217) of the differential equation

$$\frac{\partial^2 u}{\partial y^2} - x\frac{\partial^2 u}{\partial x^2} + (x-1)\frac{\partial u}{\partial x} = 0, \text{ with } u(x,0) = u(x,\pi) = 0.$$

Problem 1) may come in handy for notation.

3) Use separation of variables to find product solutions of

$$\frac{\partial^2 u}{\partial t^2} = -\frac{\partial^4 u}{\partial x^4}, \text{ with } u(0,t) = u(1,t) = u_{xx}(0,t) = u_{xx}(1,t) = 0$$

Hint one of the reduced equations should be 4th order linear so Appendix A.2 may help.