

## Math 415/515 Homework 4

**Directions:** NEATLY write all solutions on your own paper. Be detailed in your answers.

1) Use Residue theory to find the inverse Fourier transform of  $F(w) = \frac{2}{1+w^2}$ . That is evaluate  $\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2}{1+w^2} e^{iwx} dw$ . Hint: Consider two cases. 1)  $x > 0$  and 2)  $x < 0$ .

2) Find the Linear Fractional Transformation that send the three points  $z_1 = i$ ,  $z_2 = 0$ ,  $z_3 = -1$  to  $w_1 = 0$ ,  $w_2 = i + 1$ ,  $w_3 = 1$ . Let Maple or something else do the algebra.

3) Verify that the conformal map in 2) sends the circle  $|z| = 1$  to the real line and the circle  $|z - \frac{1}{2}| = \frac{1}{2}$  to the line  $v = 1$ .

4) Solve the Dirichlet problem  $\nabla^2 \phi = 0$  for  $-\infty < x < \infty$ ,  $0 < y < 1$  with  $\phi(x, 0) = 2$  and  $\phi(x, 1) = 5$ . Hint: Your answer will be independent of  $x$ .

5) Use 2-4 mapping to solve the  $\nabla^2 \Phi = 0$  on the domain between the two circles  $|z| = 1$  to the real line and the circle  $|z - \frac{1}{2}| = \frac{1}{2}$  where  $\Phi(x, y) = 2$  on  $x^2 + y^2 = 1$  and  $\Phi(x, y) = 5$  on  $(x - 1/2)^2 + y^2 = 1/4$ .

