Math 415/515 Homework 3

Directions: NEATLY write all solutions on your own paper. Be detailed in your answers.

1) Velocity fields of fluid flows are frequently given by complex functions with circulation defined as $circulation = Re(\oint_C \overline{f(z)})dz$. Compute the circulation for the velocity field of the fluid flow given by $f(z) = \dfrac{2+i}{z}$, where C is the circle of radius one centered at 0.

2)Use the Residue Theorem and Cauchy P.V. to evaluate

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2(x^2+2)} dx.$$

3) Use residue theory to show $\displaystyle\int_0^{2\pi} \frac{\cos(3\theta)}{5-4\cos(\theta)}\, d\theta = \frac{\pi}{12}$.

4) The Laplace Transform

$$F(s) = \mathcal{L}(f) = \int_0^{\infty} e^{-st} f(t) dt$$

is a powerful tool in mathematics, statistics and Physics. The inverse can be difficult to compute over the reals but if $F(s)$ satisfies mild decay conditions it can be computed by

$$\mathcal{L}^{-1}(F) = \sum_k Res(e^{st}F(s), s_k)$$

Use this formula to compute $\mathcal{L}^{-1}(F)$ where $F(s) = \dfrac{s}{(s^2+1)^2}$

5)Suppose $f(z)$ has a simple pole at a on the real axis. If C_r is the contour defined by $z = a + re^{i\theta}$, $0 \le \theta \le \pi$ show,

$$\lim_{r\to 0} \int_{C_r} f(z)dz = \pi i Res(f(z), a)$$