

Math 415/515 Homework 2

Directions: NEATLY write all solutions on your own paper. Be detailed in your answers.

1) Verify the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire plane and find its harmonic conjugate.

2) Suppose $f(x + 0i) = e^x$, $f(0) = 1$ and $f(z)$ is entire. Show that if $f'(z) = f(z)$ then $f(z) = e^z$

3) Evaluate the integral $\int_C \bar{z} dz$ where C is the path parameterized by $z(t) = t + it^2$ for $-1 \leq t \leq 1$. Can you find a different path between $z(-1)$ and $z(1)$ so the integral will have a different value? Is $f(z) = \bar{z}$ analytic? Why or why not?

4) Use Cauchy's Integral formula for derivatives to evaluate $\oint_C \frac{z+1}{z^4 - 3iz^3} dz$ where C is the circle $|z| = 1$.

5) Verify the Cauchy-Riemann equations for Polar coordinates. That is show if $f(z)$ is analytic on an open set S then $u_r = \frac{1}{r}v_\theta$ and $v_r = \frac{-1}{r}u_\theta$.