

## Math 415/515 Homework 1

**Directions:** NEATLY write all solutions on your own paper. Solutions should include details like "by the triangle inequality we get .." You may discuss the problems with other but write up your solutions on your own.

1) If  $|z| = 1$  show  $1 \leq |z^2 - 3| \leq 4$ . Draw the region (called an annulus) defined by  $2 \leq |z-3| \leq 4$  (note there is no square on the variable  $z$  this time).

2) Show that for any two complex numbers  $z_1$  and  $z_2$

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$$

3) If we define  $z_1 \times z_2 = \text{Im}(\overline{z_1}z_2)$ , show  $|z_1 \times z_2|$  is the area of the parallelogram given by the points corresponding to  $0, z_1, z_2$  and  $z_1 + z_2$ .

4) Show that for the solutions to  $z^5 = 1$  sum to zero. What about  $z^n = 1$  for any positive integer  $n$ ?

5) Show that the identity  $1 + z + z^2 + z^3 + \cdots + z^n = \frac{1 - z^{n+1}}{1 - z}$  holds for any complex number  $z$ . Plug in  $z = e^{i\theta}$  and take the real part of both sides of the expression to complete a trig identity for

$$1 + \cos(1\theta) + \cos(2\theta) + \cos(3\theta) + \cdots + \cos(n\theta) = \frac{1}{2} + \frac{\cos(n\theta) - \cos((n+1)\theta)}{2 - 2\cos(\theta)}.$$

Use the identity to compute  $1 + \cos(1) + \cos(2) + \cos(3) + \cdots + \cos(1000)$ .