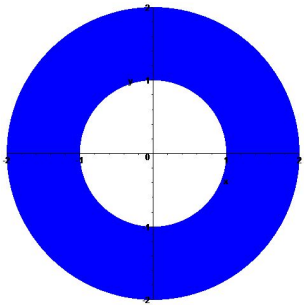
Math 515 Homework 4 Due: 12/3

Directions: NEATLY write all solutions on your own paper. Solutions should include details like what identities were used and any complicated computations. You may discuss the problems with others but write up your solutions on your own.

1) Where does the conformal mapping $w = ln(z) = ln(r) + i\theta,\ -\pi < \theta \leq \pi$ send the annulus $1 < |z| < 2$? In particular what happens when the annulus intersects the negative real axis.



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2) Find the Linear Fractional Transformation that send the three points $z_1 = i$, $z_2 = 0$ $z_3 = -1$ to $w_1 = 0$, $w_2 = i + 1$ $w_3 = 1$. Let Maple or something else do the algebra.

3) Verify that the conformal map in 2) sends the circle $|z| = 1$ to the real line and the circle $|z - \frac{1}{2}| = \frac{1}{2}$ to the line $v = 1$.

4) Solve the Dirichlet problem $\nabla^2\phi = 0$ for $-\infty < x < \infty$ $0 < y < 1$ with $\phi(x, 0) = 2$ and $\phi(x, 1) = 5$. Hint: Your answer will be independent of x.

5) Use 2-4 mapping to solve the $\nabla^2\Phi = 0$ on the domain between the two circles $|z| = 1$ to the real line and the circle $|z - \frac{1}{2}| = \frac{1}{2}$ where $\Phi(x, y) = 2$ on $x^2 + y^2 = 1$ and $\Phi(x, y) = 5$ on $(x - 1/2)^2 + y^2 = 1/4$.