Math 515 Homework 3- Due 11/17

Directions: NEATLY write all solutions on your own paper. Solutions should include details like what identities were used and any complicated computations. You may discuss the problems with other but write up your solutions on your own.

1) Use residue theory to compute $\oint \frac{2z+6}{z^2+4} dz$ where C is the circle $|z+i| == 2$

2)Use the Residue Theorem and Cauchy P.V. to evaluate

$$\int_{-\infty}^{\infty} \frac{x^2}{(x^2+1)^2(x^2+2)} dx.$$

3) Use residue theory to show $\int_0^{2\pi} \frac{\cos(3\theta)}{5-4\cos(\theta)}\, d\theta = \frac{\pi}{12}$.

4) Suppose $f(z) = \frac{g(z)}{h(z)}$ where g, h are analytic, $g(z_0) \neq 0$ and h has a zero of order 1 at z_0. Then the residue of f at z_0 is $\frac{g(z_0)}{h'(z_0)}$, i.e., $Res(f(z), z_0) = \frac{g(z_0)}{h'(z_0)}$ and use this to compute $\oint_C \tan(z)\, dz$ where C is the circle $|z| = 2$.

5) The Laplace Transform

$$F(s) = \mathcal{L}(f) = \int_0^{\infty} e^{-st} f(t) dt$$

is a powerful tool in mathematics, statistics and Physics. The inverse can be difficult to compute over the reals but if $F(s)$ satisfies mild decay conditions it can be computed by

$$\mathcal{L}^{-1}(F) = \sum_k Res(e^{st} F(s), s_k)$$

Use this formula to compute $\mathcal{L}^{-1}(F)$ where $F(s) = \frac{s}{(s^2+1)^2}$